YOU MAY NOT ENJOY MATHEMATICS

(but you do not have to hate it)

Sandoval Amui

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In this book, I offer the reader highly speculative math concepts, some of them in full disagreement with current math concepts.

In my attempt, I kept in mind an ancient saying:

"If you don't try, you won't fail,

but neither will you ever succeed."

INITIAL WORDS

In writing this book, I kept in mind the vast contingent of people, who say they hate math.

In this book, I propose some innovative (although highly polemic) math concepts, which will lead us to a different math structure. I believe the speculative character of this book makes it quite different from others.

I am aware this is a bold initiative, particularly coming from a person, who never had mathematics as his professional field of endeavor. Math is an ancient science, known as the “Queen of Sciences", and reputed to be the most difficult discipline in any academic program.

Just to prepare the reader’s mind, I say that the new math structure I propose (here named the “New Math Model”) will deliver the same useful results as the math structure in use (here called the “Traditional Math Model”) does.

Next, comes the inevitable question:

> **Why should anyone bother with the inconvenience and trouble to replace an ancient math structure, which yields useful results, by a new one, which delivers the same useful results?**

To that extent, I will present sustainable arguments to convince the reader that this new approach is worth the effort. Initially, I must say that the rationale behind theoretical explanations and the interpretation of results will be significantly different under the New Math Model.

Additionally, any person with a minimal familiarity with math knows that, in addition to the useful results, the Traditional Math Model yields strange results, as imaginary roots of equations, equations with strange roots, indeterminate forms, multiplication and division operations with positive and negative numbers and terms, and other oddities not satisfactorily explained. Current concepts often accept positive values as equivalent to absolute values, what is not always a valid approach.

It is undeniable that most students in math classes do not achieve the same performance and grades they do when attending other disciplines. In addition to poorer learning, it is no secret that most students and people in general dislike the science of math.

> **I often hear highly skilled people say they hate math.**

It is an unquestionable fact that most people in their day-to-day activities only need an understanding of elementary math, a knowledge that does not go much further beyond the four basic arithmetic operations. This fact should be enough to justify the updating of the math programs in undergraduate schools. Complex and abstract matters should be part of advanced math programs.

I dare to say that some unproven conjectures and unsolved math problems, including the so-called “Millennium Problems” (math problems the Clay Mathematics Institute offers significant money prizes to whom will solve any of them) may be improper statements, formulated on invalid premises. In other words, it is not possible to prove a conjecture or solve a problem, if said conjecture or problem does not exist.

Notwithstanding all these unquestionable odd situations, specialists keep adding new theories to the math model in force, building a giant structure over a defective foundation. Any time they find an inconsistent result, they provide strange (and clearly unacceptable) explanations, instead of questioning the fundamentals used.

> **We have to accept the obvious conclusion: there is something wrong with math!**

That conclusion is the main reason why we should open our minds, and consider a new math structure, simpler and more consistent math model, free of poorly explained odd results, less abstract and better aligned with the real world. This is an attempt to improve students’ achievements and people's appreciation when dealing with math matters.

The introduction of the new math structure fundamentals is the goal I have in mind. It may sound too presumptuous, I admit. However, it is simply a well-meaning attempt to raise and offer a different view about certain math subjects.

> **If you don’t try, you won’t fail. If you do try, you may fail anyway, but you may succeed.**

In the following pages of this book, I will present arguments, which support my own answer to the question made above, as well as the guidelines to follow to achieve that target.

In addition to questioning some math results and their odd explanations, I accepted a different interpretation for some math fundamentals, as the concept of numbers, and introduced a Fundamental Axiom of Mathematics and an Alternative Cartesian System. As a by-product, I made a clear distinction between positive values and absolute values. I also took the opportunity to review other traditional math subjects, such as Pythagoras’ Theorem and Fermat’s Conjecture (Fermat’s Last Theorem after Andrew Wiles), two math subjects, which, in my view, we cannot treat as pure algebraic matters.

I must emphasize that I limited the scope of this book to the analysis of math fundamentals. It is not my intention to extend the application of the innovative concepts of the New Math Model to the realm of pure mathematics, or to the math segments of applicable math, except to a few elementary illustrative examples, with the purpose to test their feasibility. Nevertheless, to the extent I could verify, they worked perfectly.

It is also relevant to emphasize that the introduction of innovative concepts to the science of math will not be sufficient to improve students' achievements when attending courses on this science. It is also necessary to review and update the didactic approach adopted by academic programs. The present approach, in addition to dealing with questionable abstract subjects, emphasizes the calculation phase when solving math problems. This is a mechanical task that modern machines will perform with incomparable speed and accuracy, leaving in a secondary plane the reasoning about the problems, what they mean, which math concept to use to handle them, and how to interpret the results.

As a tool to serve other areas of the human knowledge, math deals with different matters, which obey their own laws. Under these circumstances, and without breaking its own valid rules, math must conform to the laws ruling the area it serves. For example, the letter "x" in an algebraic expression may mean the unknown value of an equation, a geometric property of a geometric figure or the abscissa of a point that belongs to a curve placed in the Cartesian graph, as well as a physical property in a scientific formula. The same letter in a similar math expression may have different meanings, what requires different treatments.

I suppose teachers do not make this concept clear to their students when teaching math classes. Perhaps, because they themselves do not have that perception. My suggestions will not solve all the inconsistencies we find when using the present math structure, but at least will improve the instrument. The new math structure will be less abstract, more accurate, and will not require odd explanations.

"Logical reasoning" is the foundation and "simplicity" is the target of my propositions. My suggestion of a simpler math structure may or may not work satisfactorily, within reasonable doubt. The math structure in use simply does not (that is a certainty).

In writing this book, I kept in mind the vast contingent of people who say they hate math. If the reader is among those persons who have such unpleasant feeling against math, I am convinced that he or she would not hate the math and the teaching approach I propose in this text (at least, not as much as he or she hates the present math approach).

You may not enjoy math, but you do not have to hate it.

LIST OF SUBJECTS

CHAPTER I

THE SCIENCE OF MATH

Contrarily to popular belief, math is not a perfect and exact science. In fact, math is the science of approximate results.

Human knowledge

Human knowledge encompasses different specialized areas of studies, called "sciences". Some of these sciences created by human being have the purpose to deal with a pre-existing subject, as physics and astronomy, while others, as mathematics and statistics, work as tools to serve other areas of endeavors. In its role to support all other fields of knowledge, math works as the "scientific language", it being the reason we refer to it as the "Queen of Sciences".

It is clear that math, as a tool to serve other fields of work, and without breaking its own valid rules, must conform to the laws ruling the subject of studies of the science it provides support. It means that, contrarily to the popular understanding, when math deals with geometry, it has to obey the laws of geometry[1]. In case of conflict or disagreement between the laws applicable to math and the laws applicable to geometry, the geometry laws will always prevail.

> **Geometry is a perfect creation of Mother Nature. Math is an imperfect creation of human being.**

The condition above stated does not prevent math from treating geometry under different methods and tools, as algebra, calculus, topology, and other means, provided math does not break any natural geometric law.

Any person reasonably familiarized with elementary math knows that this area of endeavor (particularly algebra) coexists with unproven conjectures, unsolved problems, and other odd results, as imaginary numbers, strange root of equation, equation with an infinite number of roots, and the like.

Math undoubtedly has unquestionable importance and beauty, and human beings would still be living in the Chipped Stone Age if math did not exist. However, contrarily to the common sense, math is not a perfect science, not even an exact science. Math concepts and premises in use need revision and improvements. To illustrate this thought, let us recall some of these problematic subjects:

(i) In most practical cases, we deal with real numbers, which are not exact values (non-terminating decimals); it means that we often have to accept approximate results in our calculations.

1 The geometric laws found in nature, not any geometric law established by human beings.

(ii) **When we use the number "zero" in math operations, we sometimes face undefined and indeterminate forms, and other unclear situations.**

(iii) **Numbers may be positive or negative, but negative numbers do not follow the same rules applicable to positive numbers. We then have to coexist with the questionable theory of "complex numbers". Additionally, math accepts positive numbers as equivalent to absolute values.**

(iv) **The use of the Cartesian System by algebra is rather misleading, since this method does not have the widespread application as accepted under current math concepts.**

(v) **There is a list with a significant number of unproven conjectures and unsolved problems, to which the best mathematicians around the world have not found the correct responses.**

As I will demonstrate in Chapter IV, it is relevant to keep in mind that:

> **In our daily use of math, we deal with arithmetic and algebraic equalities.**

Real numbers

Math is not capable to represent in a sound way the continuity existing in nature, either in geometry or in other fields of work. The concept of real numbers, which encompasses rational and irrational numbers, is not a satisfactory approach, since it is not possible to express and handle a figure with an infinite number of decimals (non-terminating decimals). It means that math equalities are not always true statements, and math results are exact values only when input and output figures are whole numbers.

This is a serious limitation, perhaps the most important and fundamental math problem, which allows us to state that:

> **Contrarily to the popular belief, math is not a perfect and exact science. In fact, math is the science of approximate results.**

Operations with "zero"

The use of "zero" as a common number takes us to some uncomfortable situations, as undefined values, in case we try to divide any real number by zero, and indeterminate forms, as the limit of a ratio of two functions, when both functions tend to zero when approaching the desired limit. The latter, two math expressions in relation to a common point belonging to their Cartesian representations that rests on the axis of the abscissas. I will return to this subject in Appendix "A".

I wonder if we ever overcome the operational difficulties, we face when dealing with the number zero.

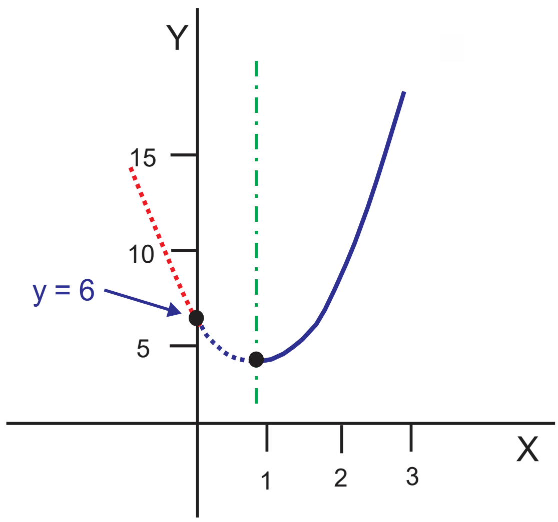
Complex numbers

The theory of complex numbers appeared to solve a non-existing problem: to find the roots of a math expression erroneously understood as a cubic equation, and which by another mistaken concept may have up to three roots. Considering the fundamentals of the Cartesian method, it becomes clear that said "cubic equation" (as any other Cartesian representation of geometric figures or random algebraic expressions) is not an equation and has no roots whatsoever.

We may place a same parabola in an infinite number of positions in a Cartesian graph, and for each position we place the same parabola there will be a corresponding, but different algebraic expression "$y = f(x)$". If in any of these infinite algebraic expressions we make the ordinate "$y = 0$", math says we create a 2^{nd}-degree equation, and said equation must have two real or imaginary roots. This is just a simple example of a questionable concept accepted by current math.

If we are not able to find a value for the independent variable "x" in the math expression of a parabola "$y = 3x2 – 5x + 6$", when the dependent variable "y" is equal to zero, it only means that said parabola, in that specific position in the Cartesian graph, does not touch the axis of the abscissas. As seen in Figure I.1, it is not necessary to accept the existence of imaginary roots[2].

Figure I.1: Parabola



Equations and Cartesian representations of random algebraic expressions or of geometric figures are two different math subjects.

The theory of complex numbers is a huge building constructed over poor quality piles, and remains standing exactly because it only exists in our imagination.

2 In Author's view, the acceptance of imaginary roots is a paradox, a self-contradictory concept.

We may disregard the theory of complex numbers, as well as all further developments relating to the same subject, including certain unproven conjectures and unsolved problems.

Under current concepts, we sometimes consider positive numbers as equivalent to absolute values, what (as I will show) is not always a valid assumption.

Use of the Cartesian System

The Cartesian System is a magnificent algebraic tool, which allows the algebraic representation of a flat or a tri-dimensional geometric figure placed in relation to a system of two or three orthogonal axes. With the use of the Cartesian System, we may represent in terms of algebra a well-known geometric figure, as an ellipse or an ellipsoid. We will be dealing with a geometric figure that exists in the real world, which obeys a geometric law, and will get a meaningful algebraic expression.

This one-to-one relationship between algebra and the Cartesian method, being a reversible relationship, also allows the opposite action: the representation of any given random algebraic expression (up to three variables) as a geometric figure drawn in the Cartesian System. Under this second approach, the randomly chosen algebraic expression may represent a geometric figure that does not exist in nature, which does not follow any geometric law, and we may be dealing with a meaningless algebraic expression.

I will explain at a later stage in this book that in both cases above, the prevailing math concepts about the use of the Cartesian System may induce us to misleading interpretations.

The Cartesian System is an algebraic method, which allows us to express in algebraic language a geometric figure placed in a system of two or three orthogonal axes, as well as the representation of a random algebraic expression as a geometric figure in the that system.

Math challenges

There are many unproven conjectures and unsolved problems defying mathematicians and non-mathematicians worldwide. Some of these conjectures and problems have been challenging the best mathematicians in the world for centuries. The most famous are the so-called Millennium Problems, as selected by the Clay Mathematics Institute in 2000. Initially seven unsolved problems considered as the most difficult math challenges. I will refer to them at a later moment in this book.

If we modify math fundamentals, we must review the meaning of unproven

conjectures and unsolved problems

Questioning concepts

I have no suggestion on how to overcome the two first difficulties previously mentioned: the impossibility of exact values with real numbers (non-terminating decimals), and the problems related to the use of zero as a common number (undefined values). I will limit my comments to the third subject, the concept of number and related matters, as the meaning of letters in algebraic expressions, and to the fourth subject, the use of the Cartesian System, and the difference between positive values and absolute values. I also imagine that a different concept of number and letters in math expressions may shed some light on the fifth subject, the unproven conjectures and unsolved problems.

Math fundamentals currently in use are clearly inadequate and need improvements. Therefore, all further developments built in accordance with inconsistent premises and concepts often originate mistaken results. That is why I suggest some new math fundamentals, which will lead us to a different math structure.

However, before introducing the new math fundamentals and their effects on math applications, it is convenient do review some preliminary matters. It is also necessary to clarify that, for simplicity, in this book I will use the wordings “math expression”, “algebraic expression” and “math equality” as general terms to refer to any math proposition, as equalities, equations, formulas and Cartesian representations of geometric figures.

We may face a misleading result when we formulate a problem based upon inconsistent premises. We may also mistakenly use an inappropriate math concept when trying to solve a problem, even a consistent problem. In both cases, we may get strange results or end up in a deadlock. Under these events, we should not blame math for odd situations, we should question the premises we accepted and the math tools and methods we used, with the purpose to obtain consistent and acceptable results.

It is my view that some unproven conjectures, unsolved problems, as well as other unclear situations still existing are math matters affected by the previously referred reasons of inconsistent premises or inappropriate approaches. Some of the Millennium Problems, which have been defying the best mathematicians for centuries, may be examples of invalid propositions.

These intrinsic math difficulties together with the inadequate teaching methodology we often see in math classes, which emphasizes the calculation phase when solving math problems (the least important, since calculators and computers may perform said tasks with incomparable speed and accuracy), probably explain why most people say they hate math. It seems we have been adopting a poor approach to teach students an inconsistent discipline.

In writing this book, I looked for the answers to the following questions:

(i) What are the causes of all these math oddities and inconsistencies?

(ii) What conceptual improvements we should adopt to overcome said oddities and inconsistencies?

(iii) How to improve students' achievements in math courses (undoubtedly considered by students to be the most difficult discipline), and, as a by-product, increase people's appreciation for the science of math?

CHAPTER II

MATH AS THE SCIENTIFIC LANGUAGE

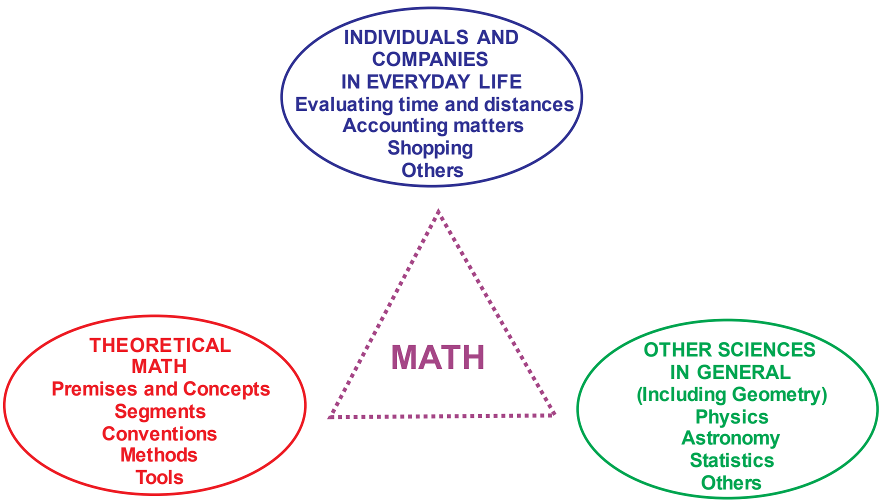
Math is a tool, the scientific language, which supports human activities.

Applications

As a scientific language, math serves people, companies and sciences in many ways in everyday life. In its role, math uses the same framework under different rules and for distinct purposes, but the employment of a same foundation (which by itself coexists with inconsistencies), under different rules and for distinct purposes, originates some conceptual confusion.

Figure II.1 summarizes the three main groups of uses of math, illustrating how broad and complex said uses are. Individuals and companies use math in their daily life under an uncountable ways, going from simple shopping or bank balance checking to highly important international business. All sciences need math to perform their tasks, either describing the behavior of phenomena, as physics and astronomy, or as auxiliary sciences, as statistics and accounting. Finally, math works for itself, by developing new theories and methods, with the purpose to improve its performance and applications.

Figure II.1: Math overall applications



The analysis of the possible and almost infinite ways to use the science of math in our day-to-day activities is not the target of this book. Instead, I intend to discuss the consistency of some fundamentals presently accepted by math, as well as their effects on specific math applications.

Segments and methods

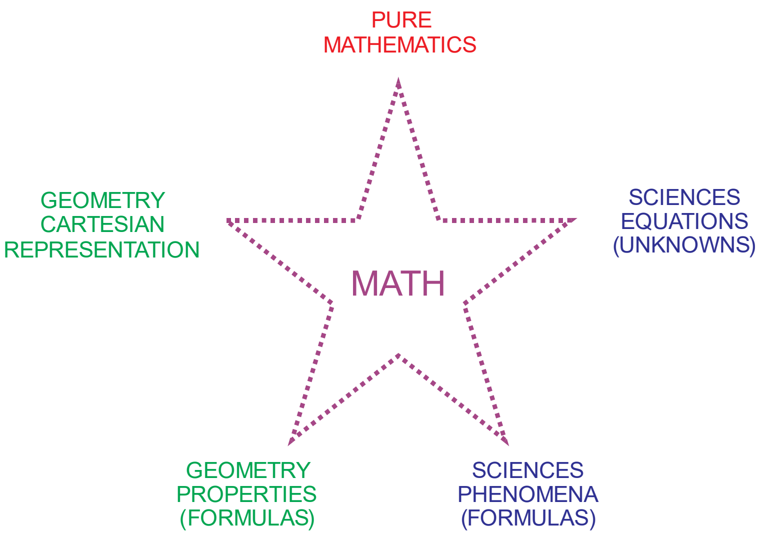
Starting from very basic elements, as numbers, math grows by means of segments, in a manner that any previous segment is the foundation for the new ones, in a development sequence. It means that any imperfection accepted in any of these basic segments will propagate to the subsequent segments, it being an aspect of the interest of this book. At present, and roughly speaking, the science of math contemplates arithmetic, algebra, calculus, probability and many other segments.

There is a widely spread understanding that geometry is a segment of math, but that is an improper approach. In fact, there is a strong interaction between math and geometry, in a manner that even specialists mix and confuse the two sciences, what results in joint subareas as algebraic geometry, analytic geometry, fractal geometry, topology and the like. Nevertheless, geometry is an independent science, which has its own subject of studies: the geometric figures.

Practical uses of math

We see some particular math applications of the interest of this book in Figure II.2.

Figure II.2: Scientific uses of math



On purpose, and contrarily to the common understanding, I showed “pure math” as an application of math. The reason behind that view is my understanding that new math theories, methods and tools must serve the real world.

Scientific formulas

In some cases, we deal with formulas and a consistent unit system to define the behavior of natural phenomena and events, as we do in physics. As an example of math use in sciences, as established by the English physicist and mathematician

Isaac Newton, the law of universal gravitation states “The attraction force “f_G” between two masses “m_1” and “m_2” is directly proportional to the product of the masses and indirectly proportional to the square of the distance “d” between the centers of the masses”. In math form:

$$f_G = [k(m_1)(m_2)]/d^2$$

In sciences, we may deal with a group of math expressions as Maxwell’s formulas relating to electromagnetism, commonly named equations. We may use other math tools as calculus when solving scientific questions.

Properties of geometric figures

Math serves geometry with “formulas” to quantify the properties of the geometric figures (length, areas and volumes). The volume of a sphere, mathematically expressed as a function of its radius “r” is:

$$V = (4/3)\pi r^3$$

Given the sphere radius, its volume will be the same, no matter the sphere is in my mind, as a drawing in a piece of paper or as a real soccer ball over a table. The sphere volume is a geometric property, and its value is solely dependent on the sphere radius.

Cartesian representation of geometric figures

Math also interacts with geometry with the help of a Cartesian System of orthogonal axes to represent “geometric figures in the algebraic language” and “vice-versa”. A different application in connection with the same area of studies: geometry. The Cartesian System is a conventional mean to join geometry and algebra.

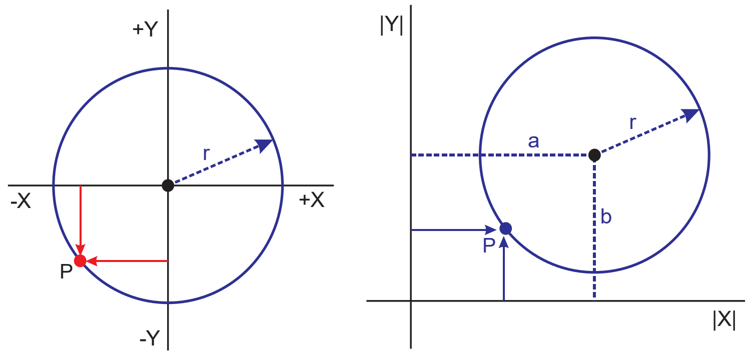
The algebraic representation of a sphere of radius “r” with the use of the Cartesian System is:

$$x^2 + y^2 + z^2 = r^2$$

Contrarily to a geometric property of a geometric figure, as the area of a circumference, which remains constant for a same geometric figure, the algebraic expression will be different if we place the same geometric figure in a different position in the Cartesian System, as illustrated in Figures II.3(a) and II.3(b):

Figure II.3: Cartesian representation of a circumference

(a) (b)



$x^2 + y^2 = r^2$ $(x - a)^2 + (y - b)^2 = r^2$

When dealing with geometry, we use different algebraic expressions, under different rules to: (i) determine the properties of geometric figures with "formulas"; and (ii) represent geometric figures placed in the Cartesian System in terms of algebra.

As a reversible mean to join algebra and geometry, the Cartesian method also allows the opposite action: to represent some random algebraic expression as a geometric figure in the Cartesian System, a subject I will discuss at a later moment in this book.

Equations

"To solve equations and systems of equations" is a common math subject in any school program and evaluation tests. Under the present math concepts, we may have equations with one or more than one unknown, and systems of "n" equations with "n" unknowns. The algebraic expressions assumed as being equations may be of any algebraic degree. Teachers request students to "solve" certain highly complicate group of algebraic expressions (meaning finding the acceptable values of common unknowns that match the equalities imposed to all of them), without explaining why they are doing that and how to interpret the results.

Contrarily to the common understanding, and as I will explain, an equation is a mean to give an answer to a pre-formulated question when it builds and solves certain algebraic equalities of a single unknown[3]. The unknown value (x) is the answer to the formulated question. As an example:

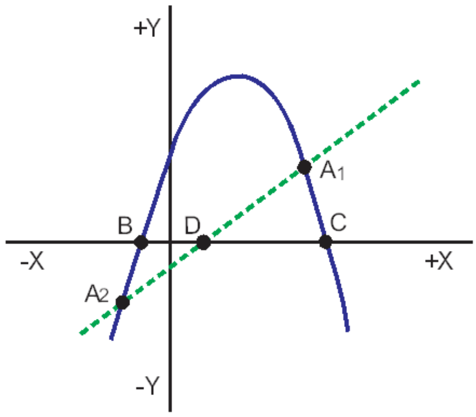
$$\pm a_1x \pm b_1 = \pm a_2x \pm b_2$$

3 This concept is not mine; I simply accepted it as a valid concept.

In Figure II.4, we see what math understands as a system of two equations:

$$\begin{cases} y = a_1x + b_1 \\ y = a_2x^2 + b_2x + c_2 \end{cases}$$

Figure II.4: Intersection of a parabola by a straight line



These two expressions are not equations and do not form a system of equations. They are the Cartesian representations of a straight line and of a parabola. The two figures may have common points or not. The straight line may intercept the parabola in two ways: in two points "A_1" and "A_2", or be tangent to it in a single point. The two figures may also not touch each other. Any common point that may occur will depend on the positions of the two geometric figures in the Cartesian graph. Except in case of a previous assumption, common points (as "A_1" and "A_2"), as well as the points where the two geometric figures intercept the axis of the abscissas (as "B", "C" and "D") do not have any special meaning.

> **Equations and algebraic representations of geometric figures in the Cartesian System are two different applications of algebraic expressions (equalities).**

I already mentioned that we have groups of scientific formulas used to describe in the mathematical language the behavior of phenomena other sciences deal with, as physics, astronomy and others, which the New Math Model interprets as "formulas", and not as "systems of equations".

Pure mathematics

Finally, at a higher level, as in research and advanced math courses, the math segment called "pure mathematics" takes care of abstract and imaginary matters, which, in many cases, do not aim at any immediate practical application.

The point I want to stress refers to the use of possible mistaken assumption or a defective foundation in the development of advanced math tools. If such is the case, the outcomes will suffer the prejudices of that mistaken assumption or defective foundation.

Each one of the different applications of math relies on its own fundamentals, a clarification the students are not aware of when taking math classes. They are not aware that formulas (in geometry or other sciences), equations, and Cartesian representations of geometric figures are three different uses of math. Just to illustrate the point, Cartesian representations of geometric figures do not create equations nor systems of equations. A polynomial made equal to zero is not an equation. There is no equation with imaginary or strange roots. In brief:

> **Algebra does not have the widespread field of work and applications as accepted under current math concepts.**

Arithmetic deals with numbers and gives us a photo, an instantaneous picture of a static situation. Algebra allows us to work with numbers and letters and gives us a film, a continuous image of an event.

Advanced math deals with geometry under different approaches, distinct math specialties of no interest of a common person, as algebraic geometry, differential geometry, projective geometry, and the like. In any case, even when dealing with these complex matters, the laws of geometry cannot be broken. We have a unique geometry, an independent area of human knowledge with a proper subject of studies (the geometric figures), which interacts with math as other sciences do.

> **When serving other areas of human knowledge, math must comply with the laws ruling the respective subject of studies of these other areas. Geometry is not a branch of mathematics, but an independent science, which deals with the geometric figures under its own rules.**

Puzzles

When attending math courses and taking examinations, students may face some difficult questions, including ambiguous arithmetic and algebraic exercises. Any reader may find myriads of these kind of puzzles on the internet. It takes math skills and knowledge to deal with these puzzles.

For a person interested in math, these puzzles may be a brain boosting exercise, or even a fun hobby, but for the great majority of common people they are nothing else but meaningless questions and unpleasant waste of time.

In writing this book, I envisaged a less abstract science, more in line with the real world, and a teaching approach consistent with the needs of common people. An attempt to improve students' academic performance and to minimize the rejection most people feel towards math matters.

CHAPTER III

NUMBERS

Numbers are neither positive nor negative math elements, but absolute values. Complex numbers do not exist.

Basic element of mathematics

The concept of number is the primary element of math fundamentals, the foundation over which mathematicians have been building the math structure currently in use. Said math structure relies on the particular premise that there are positive and negative numbers, but they follow different operating rules.

Additionally, current math accepts that positive numbers are equivalent to absolute values, what is not always a valid approach.

Math concepts in use consider that letters in any algebraic expression (representing either constants or variables) are also positive or negative elements, since there are hidden numbers behind said letters.

The concept that numbers and letters are positive or negative elements requires a "rule of signs[4]" to perform arithmetic and algebraic operations with positive and negative terms, either isolated numbers or letters, or a combination of numbers with letters. In this text, I refer to the math structure presently in force as the "Traditional Math Model".

As a speculative and philosophical approach, I want to bring into discussion a divergent view, of two alternative math structures that could result if we consider different concepts of numbers and letters representing numbers in math expressions. I briefly mentioned these alternative concepts in a previous work.

Alternative math structures

As a "first alternative math structure", numbers would still be positive or negative, but subject to another rule of signs (whatever said rule[5]), since it would still be necessary to perform arithmetic and algebraic operations with positive and negative algebraic terms. I will just make some few comments on this alternative[6], because I imagine that said math structure, if actually developed, would be as much or even more complicated than the Traditional Math Model.

4 The well-known rule about the resulting sign when performing multiplication and/or division operations with positive and negative numbers, which states that operations with numbers (or terms) of equal signs, yield a result of positive sign, while operations with numbers (or terms) with different signs, yield a result of negative sign.
5 Since (to the Author's knowledge) that math structure does not exist, that rule does not exist either.
6 Under the speculative assumption that a same operating rule would apply to both positive and negative numbers.

As a “second alternative math structure”, here named “New Math Model”, I adopted the concept that numbers are neutral elements[7] , and no rule of signs would exist, since all arithmetic and algebraic operations would deal with absolute values (modules). In order to be consistent, under that math structure, letters forming terms of algebraic expressions are modules too, neither positive nor negative elements.

The assumption will take us to a significantly different math structure. In a previous paper, I named this conceptual approach “New Fundamental Axiom of Algebra”. In this book, I refer to it as “Fundamental Axiom of Mathematics”. It is interesting to point out that things in nature, as currencies, students, cars and the like, except by convention (as credits and debts, for instance), are neutral elements.

It is important to clarify that, in spite of considering numbers and letters in algebraic expressions as absolute values, the Fundamental Axiom of Mathematics contemplates the use of numbers and letters commanded by the plus (+) and the minus (–) signs, but with the sole meaning of addition and subtraction operations of terms, which are components of math expressions.

> **Whatever the operation we perform, including multiplication, division, power or rooting, the plus (+) and the minus (-) signs solely indicate algebraic sums (addition and subtraction).**

I am not a mathematician and what I think certainly has questionable value, but in my opinion the “second alternative math structure” (the New Math Model), if developed and adopted, would be much simpler and would yield more accurate results than the math structure accepted under the Traditional Math Model.

Current math structure coexists with oddities, inconsistencies and other unclear matters, but in spite of that, and since its first steps, subsequent math theories appeared based upon the same basic premises. Professionals of this field of endeavor have accepted poor explanations, as imaginary numbers, imaginary and strange roots of equations, rule of signs and similar oddities, instead of facing and solving these problematic matters.

It is relevant to stress that the sole basic premise that will be different in this speculative work is the understanding of what isolated numbers and numbers behind letters in algebraic expressions are, what they mean, and how we must treat them in dealing with arithmetic and algebraic operations. A distinct understanding about numbers will take us to a significantly different math structure, which aims at the same goal: to serve other sciences as the universal scientific language.

The Traditional Math Model in use works satisfactorily when dealing with the real world matters, as the properties of geometric figures (lengths, areas and volumes), the behavior of scientific natural phenomena (physics, astronomy and others), and

7 This concept is not mine; I simply accepted it as the best approach to follow.

math segments which are not dependent upon certain arithmetic and algebraic concepts currently in use (probability and combinatory analysis). However, when dealing with other math matters, particularly algebraic expressions, the results are rather disappointing, since said model coexists with some poorly explained results.

For thousands of years we have been building a giant scientific structure called mathematics. The concept of number is the fundamental element, the primary support for such structure, which begins with arithmetic, followed by algebra, and some further developments of this science. In spite of so long development time lapse, we still face math oddities, poorly explained questions, unproven conjectures, unsolved problems, indeterminate forms, and other difficulties. The development of any further math theory moves along with the same unclear concepts and results.

In the development of this giant scientific structure, it seems we are relying on a poor and weak foundation, and the way math currently understands isolated numbers and numbers represented by letters in math equalities may be the weakest pile of said math foundation.

It would be more appropriate to look for alternative concepts, which may improve our knowledge about the science of math, instead of simply accepting extravagant explanations for those oddities and inconsistencies, as imaginary numbers, equations with strange roots, rule of signs, and similar fallacies.

Following this reasoning, I will discuss below different lines of thought, or approaches, regarding the understanding and uses of numbers and letters in arithmetic and algebraic expressions.

I believe there are three different ways to understand numbers and letters that represent hidden numbers in any algebraic expression, three different concepts. As a result, there would be three distinct math structures supported by said three approaches, and I will briefly comment on the possible outcomes of adopting each one of these alternative math models. However, before talking about these three ways to work with numbers, it is necessary to introduce some additional innovative, although polemic, math concepts.

CHAPTER IV

ALGEBRA AND THE CARTESIAN SYSTEM

Current concepts guide us to misleading understandings about the use of the Cartesian System by algebra.

Interaction between algebra and geometry

One of the reasons, I suppose, why students face great difficulty when dealing with math concepts, and most people say they hate math is the misleading use of the Cartesian System by algebra.

Algebra allows us to perform operations with numbers and letters, and we must handle algebraic expressions under a similar way we handle arithmetic expressions. When dealing with arithmetic expressions we only have numbers, an instantaneous picture (as a photograph), while an algebraic expression allow us to deal with numbers and letters, a continuous picture (as a film).

The Cartesian System is a method, which allows the interaction between algebra and geometry, a particular tool at the service of algebra and geometry. Whenever feasible, we may use this method to either express in terms of algebra a geometric figure placed in the Cartesian System or, contrarily, given a random algebraic expression, represent it as a geometric figure in the same Cartesian System. The latter (except in case of known geometric figures) does not, necessarily, has meaning or useful application, since we may be dealing with ludic exercises.

This System employs three orthogonal axes, it being two axes for flat geometric figures, and three axes for three-dimensional geometric figures. It means that we cannot use the Cartesian System to handle algebraic expressions with more than three variables. This type of algebraic expressions may appear in scientific formulas or as algebraic abstractions.

Principle of Cartesian coordinates

Geometry only accepts up to three dimensions, or no more than 3rd-order algebraic expressions.

Figure IV.1 shows how to determine the coordinates of a point belonging to a geometric figure with the help of the Cartesian System, either “P_P” in a plane or “P_S” is space, by applying Pythagoras’ Theorem[8].

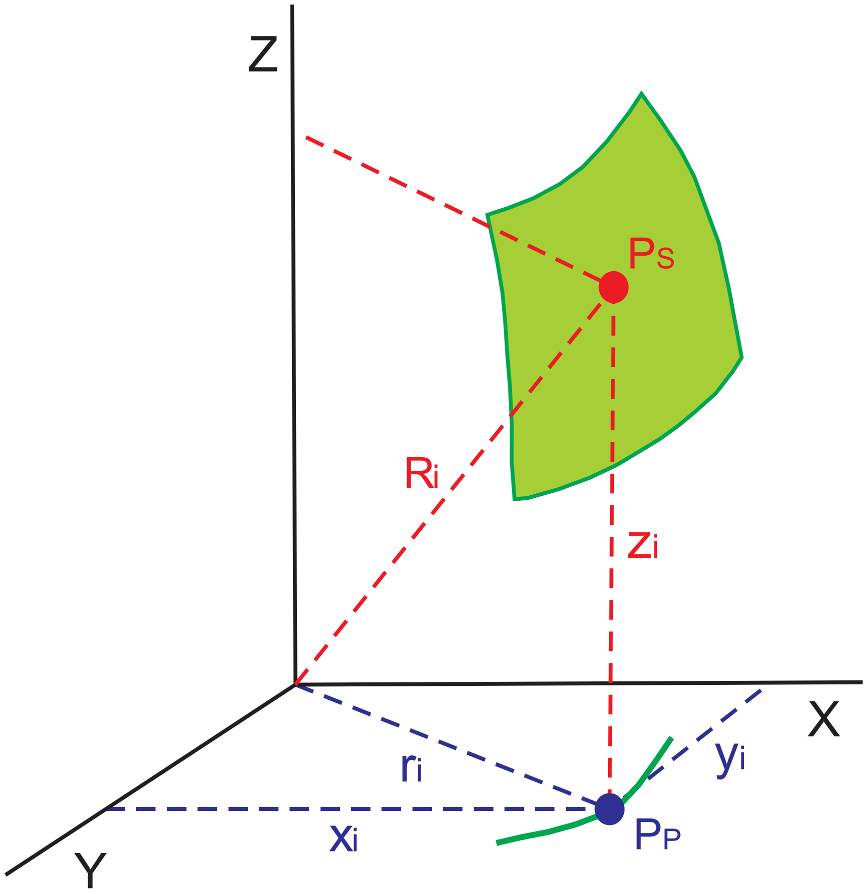
In case of more than three dimensions, we enter the fantasy realm.

8 I will demonstrate in Chapter X that, contrarily to the common understanding, the Theorem of Pythagoras is not a property of the right triangle, but a property of the circumference.

$(r_i)2 = (x_i)^2 + (y_i)^2$

$(R_i)^2 = (r_i)^2 + (z_i)^2$

Figure IV.1: Cartesian geometric dimensions



Equalities

I already stated that, in math, we deal with arithmetic and algebraic equalities. It means that an arithmetic or an algebraic expression may have meaning only when we use the equality symbol (=), separating the terms of the given math expression into two sides of the equality sign. Otherwise, the arithmetic or a math expression has no meaning or use.

To clarify this point, we may say that the following arithmetic and algebraic expressions do not have meaning or use for practical purposes:

$– 3 + 5 – 7 + 4 – 1$

$2a – 3x + ax – 3$

However, if we use the equality (=) sign, and split the terms of the math expression into two sides, we may have a workable arithmetic or algebraic expression, as follows:

$A = – 3 + 5 – 7 + 4 – 1$

In the above example, we deal with an arithmetic sum, which tells us to put together two groups of numbers, in order to find the difference between the two totals, one group commanded by the plus (+) sign, other group commanded by the minus (–) sign[9].

Similarly, in the example below, if we use the equality (=) sign, and assign a value to the constant "a", we may have an equation:

$$2a - 3x = ax - 3$$

In the above algebraic expression, "x" is the unique unknown, and its value must have a meaning, as the answer to a previous question, since this is the rationale behind the equation concept.

Contrarily, if we do not see the equality sign, we do not know how to move ahead, since we will be dealing with meaningless math expressions. As a result, in order to have meaning and be useful, math expressions require the equality (=) sign.

That is the reason why I will use the word "equality" to refer to a workable arithmetic or algebraic expression.

In practice, we deal with math equalities.

In both sides of the equality, the terms forming the math expression are commanded by either the plus (+) sign or by the minus (–) sign. Under a universally accepted convention, we may omit the plus (+) sign when placed in front of an isolated term in one side or in front of the first term in any side of the math equality (Traditional Math Model).

In the next Chapter, I will show and explain why said convention may yield two types of equalities: proper (true) and improper (false) equalities, it being the reason why the New Math Model requires that we keep in mind that, even omitted in the equality, the plus (+) sign does exist.

Typical and modified geometric figures

To understand the use of the Cartesian System as a tool to serve algebra and geometry, as well as the limitations of this method, it is necessary to discuss and distinguish the types of geometric figures we find in nature and those we may consider under the algebra realm. For the time being, I will remain within the concepts of the Traditional Math Model.

Current math concepts state that the algebraic degree of an algebraic expression corresponds to the sum of the exponents of the variables in the term of the highest

9 I kept the convention under which we may omit the positive sign (+) in front of a first term of each side of an equality expression. I will show we can omit but not ignore that plus (+) sign.

sum of exponents. Under said definition, it is possible to have algebraic expressions with an unlimited degree. Moreover, the degree means dimensions, as if there existed extra dimensions (above the 3rd-degree math expression).

I proposed a new approach in disagreement with the traditional understanding regarding the meaning of "algebraic degree". In accordance with the New Math Model, based on the laws of geometry, we have three dimensions[10], corresponding to the integer exponents "1", "2" and "3" in the algebraic expressions (dealing with length, areas and volumes). Obviously, the Cartesian System does not accept more than three variables.

In any algebraic expression:

(i) The "number of variables" only indicates if we are dealing with a flat geometric figure (two variables, "x" and "y") or a spatial geometric figure (three variables, "x", "y" and "z"); that number determines the "geometric order" of math expressions. We use algebraic expressions with more than three variables in connection with sciences (formulas) or as ludic algebraic exercises (imaginary field).

(ii) The "variable exponents" (the "algebraic degree" of math expressions) inform if we deal with a typical geometric figure (follows a geometric law and a math expression) or with a modified geometric figure (only follows a math expression). Typical geometric figures exhibit exponents of "1" and/or "2".

We see below the math expressions of some typical open and closed geometric figures, in their simplest Cartesian versions (traditional written form):

Straight line:

$$y = ax + b$$

Parabola:

$$y = ax^2 + b$$

Circumference:

$$x^2 + y^2 = r^2$$

Sphere:

$$x^2 + y^2 + z^2 = r^2$$

I will show that modified geometric figures, either open or closed, will occur if we alter the exponent of the variables in the math expression of a typical geometric figure,

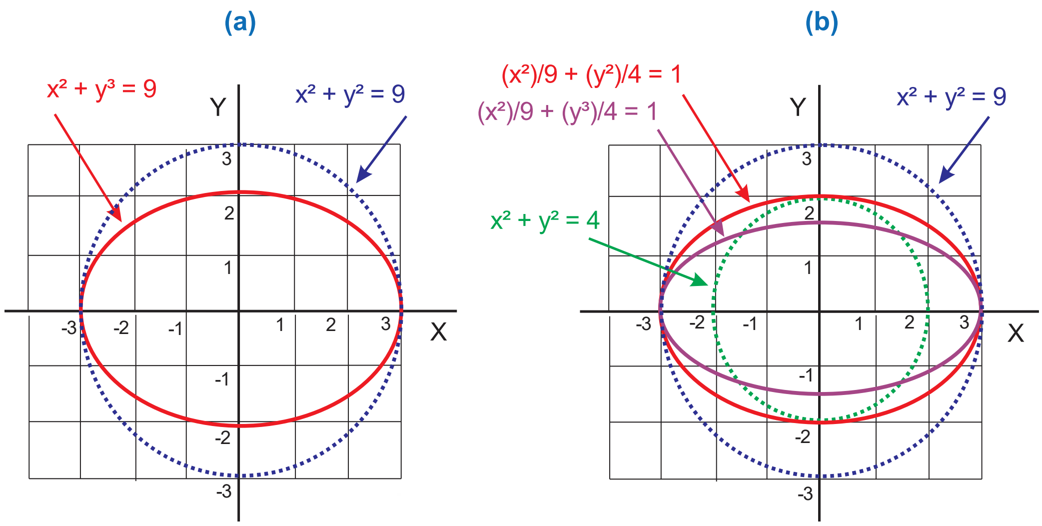
10 Unless we consider the effect of time, as in the case of the effect of yeast on cake growth.

and the resulting geometric figure may or may not exist in nature.

Under the New Math Model, in math expressions, the number of variables tells us the “geometric order” of the geometric figure considered, while the exponents of those variables give us a different meaning of the “algebraic degree”. It is also important to mention that the modification of geometric figures I refer in this book is a kind of modification that affects the geometric properties of the figure, not a single displacement in relation to the Cartesian System of axes, or a different view due to a perspective angle.

The illustration in Figure IV.2(a) shows a circumference, a typical geometric figure, which follows a geometric law, as well as a math expression ($x^2 + y^2 = r^2$). The same illustration also shows a true oval-shaped form, which follows an altered math expression ($x^2 + y^3 = r^2$), but not a geometric law, it being a modified geometric figure (not an ellipse), which results from the alteration of the math expression representing the circumference.

Figure IV.2: Typical and modified geometric figures



We see an internal oval-shaped form because we increased the exponent of one of the variables and made it greater than “2” (equal to “3”). We would obtain external oval-shaped forms in case we decrease said exponent and make it less than “2”. The example shows whole numbers, but the effects would be the same with fractional figures.

A similar effect would occur if we alter the exponents of the math expression of an ellipse (also a typical geometric figure that follows a geometric law and a math expression), as we see in Figure IV.2(b). However, the oval-shaped forms generated by altering the math expressions of a circumference or of an ellipse would not be true ellipses, because they do not obey any geometric law, but only follow an altered math expression.

To illustrate and emphasize the difference between typical and modified geometric figures, we may state:

> **There are three types of oval-shaped forms: (i) a geometric figure that follows a geometric law and a math expression (as the "ellipse"); (ii) a geometric figure that follows a math expression, only (as a "true oval-shaped form"); and (iii) a geometric figure that follows neither a geometric law nor a math expression (as an "ovoid").**

Cassini's oval[11] is a flat curve formed by the points in a manner that the product of the distances to two fixed points is constant, differently from the definition of an ellipse, which states that the sum of the distance to two fixed points is constant. A Cassini oval is a typical geometric figure, since it obeys a geometric law and an algebraic expression.

Under the suggested approach, the exponents of the variables in the math expressions of typical geometric figures are equal to "1" and/or "2" (simple application of Pythagoras' Theorem). The exponents of the variables relating to modified geometric figures may take any value, either integer or fractional figures. As an example, I will show in the next section that the so-called "elliptical curve" ($y^2 = x^3 – x + 1$) is, in my view, a "modified parabola".

I will not discuss cycloids, spirals, catenaries and other flat or three-dimensional geometric figures of few or no interest at all for the purposes of this text.

Briefly, in any algebraic expression:

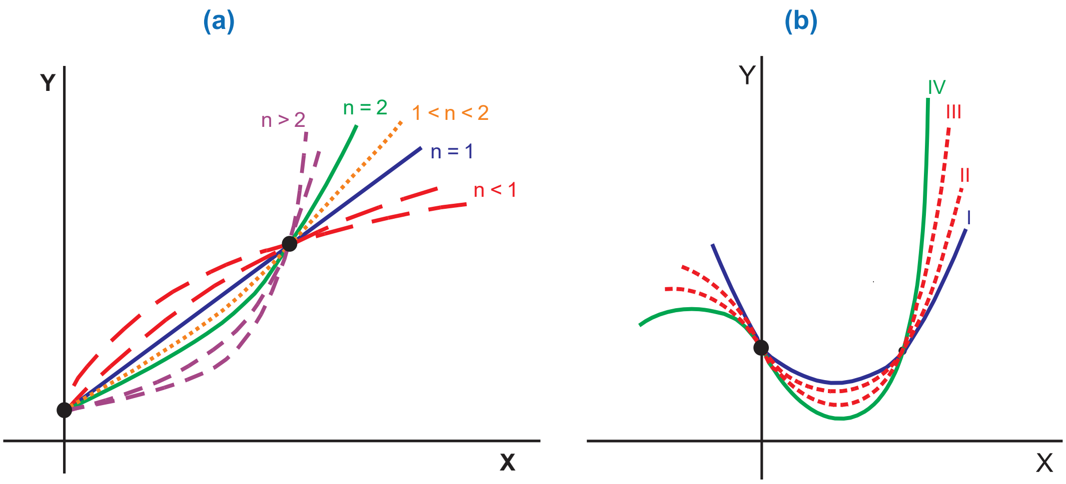
> **The number of variables informs its "geometric order", which cannot be greater than "3" in the real world (except in case of scientific formulas or dilettante exercises).**
>
> **The exponents of the variables, which represent its "algebraic degree", inform if we deal with a typical or with a modified geometric figure represented with the help of the Cartesian System.**

Polynomials, a family of flat and open curves

There is a third type of geometric figures represented by a general polynomial, which may be a typical or a modified geometric figure, and encompasses a family of flat and open geometric figures, as we see in Figure IV.3. The above-referred elliptical curve, as well as straight lines, parabolas and many others, are components of this family of geometric curves.

11 A curve named after Giovanni Domenico Cassini, who studied this geometric figure in 1680.

Figure IV.3: Exponent effects and modified geometric figures



Books, papers and other math publications state that "polynomials" represent a very important math subject, with many relevant practical applications[12]. This is the concept behind the Traditional Math Model.

Contrarily to the general belief, according to New Math Model, and with the exception of ludic math exercises, a classical polynomial[13] has no importance at all, exactly because it has no relevant practical applications, at least with respect to the day-to-day use of math by any person.

> **A common polynomial is a random algebraic expression, whatever its algebraic degree.**

To make it clear, and as examples of families of modified geometric figures, let us assume the algebraic expression "$y = \pm ax^n \pm b$", and let the "n" exponent to assume different values, as seen in Figure IV.3(a). When "$n = 1$" we have the math expression of a straight line, and for "$n = 2$" we see a parabola formula, as particular cases. For any other values for "n", either less than "1", between "1" and "2" or greater than "2", we obtain curved lines that move away from the straight line or from the parabola. These other curved lines mean nothing, in math or in the real world.

Figure IV.3(b) exhibits the well-known math expression of a parabola, "$y = x^2 - x + 1$" (full line, curve I), and when the exponent of "y" increases from "1" to "2", and the exponent of "x" increases from "2" to "3", we will reach the famous math expression of the above-referred elliptical curve, "$y^2 = x^3 - x + 1$" (full line, curve IV). Figure IV.3(b) also shows intermediate curves in between the parabola and the elliptical curve.

> **A parabola is a typical geometric figure. An elliptical curve is a modified geometric figure. In my view, a modified parabola.**

12 The algebraic handling of monomials and polynomials (addition, multiplication and the like) are not practical applications, but simply ludic exercises.

13 I refer to "classical polynomials" as a math expressions of the type: $y^m = \pm a_n x^n \pm a_{n-1} x^{n-1} \pm a_{n-2} x^{n-2} \pm \ldots \pm a_1 x \pm a$.

With the purpose to clarify my concept about polynomials, let us develop and analyze the following math equality:

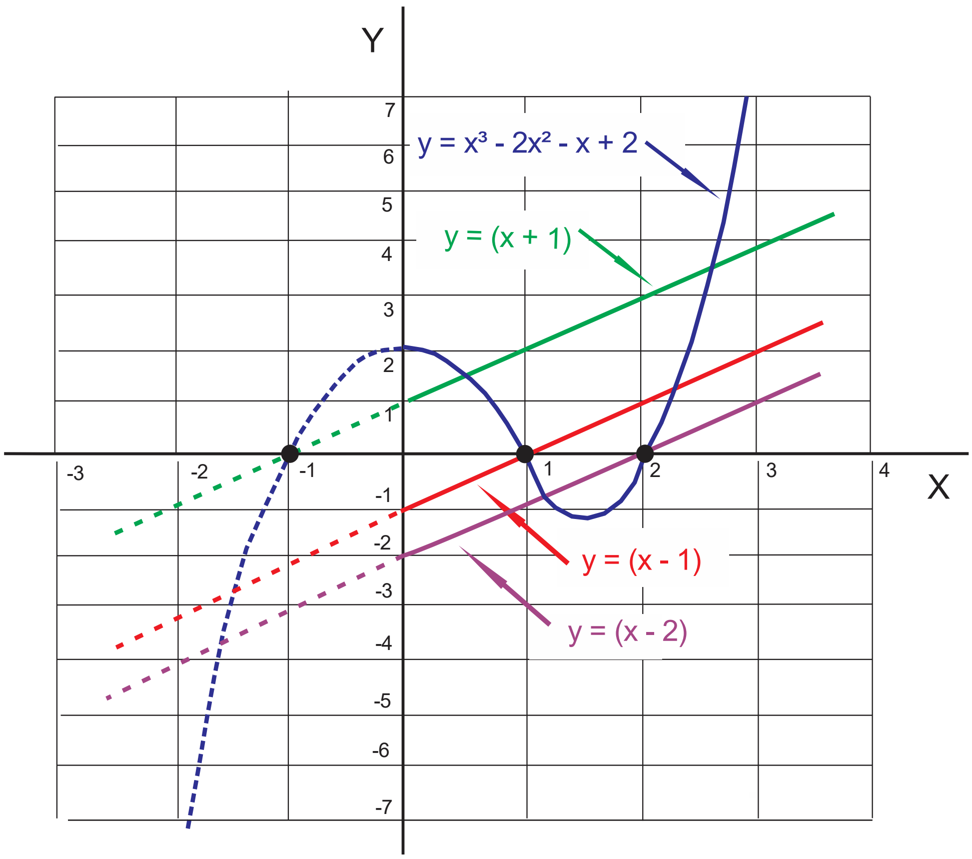
$$y = (x + 1)(x - 2)(x - 1)$$

Without considering the relevance or application that a math expression of said type may have, we face a product of three factors, which depend upon a same independent variable "x" (actually, the math expressions of three straight lines).

According to the prevailing concepts, the result of the product is "$y = x^3 - 2x^2 - x + 2$", understood as a 3rd-degree polynomial illustrated in Figure IV.4. This polynomial would have the roots "$x = -1$", "$x = +2$" and "$x = +1$", the values of the independent variable "x", which makes the polynomial equal to zero. These supposed roots result from the three factors, "($x + 1$)", "($x - 2$)" and "($x - 1$)", which originated the polynomial.

Under the approach adopted in this book, when made equal to zero ($y = 0$), the polynomial is not an equation and has no roots. With a few possible exceptions, that math expression (value of "y") does not have meaning or practical application. To mean something, the independent variable "x" must be an absolute value that represents things of the world and cannot be negative. The dependent variable "y", as an algebraic result, may assume any value, positive, negative or zero, but it only exists for "$x \geq 0$" (the continuous line in Figure IV.4).

Figure IV.4: Polynomial



Geometric law and Cartesian representation

To emphasize the meaning and use of the Cartesian System by algebra, and the conceptual difference between the prevailing approach (Traditional Math Model) and the approach here suggested (New Math Model) let us consider the two following examples:

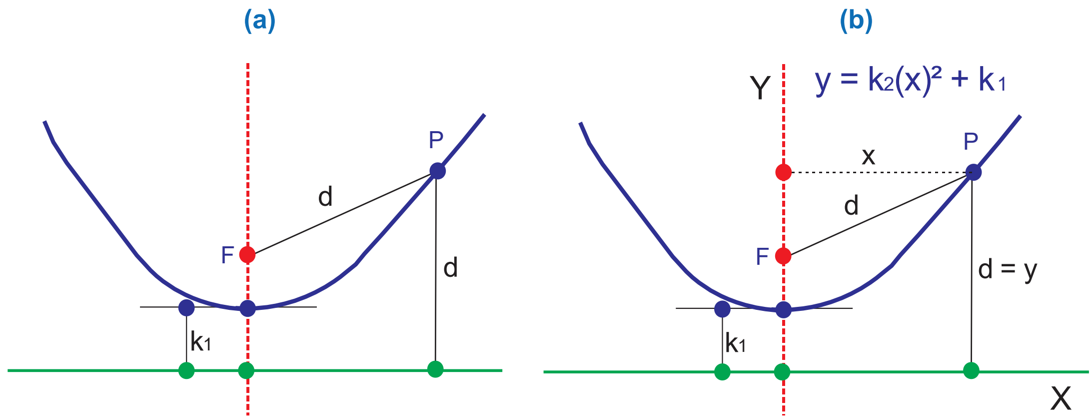
First, the well-known parabola, a typical geometric figure, entirely defined by a geometric law:

> **Parabola is a geometric curve in a plane defined with the help of a straight line (full line) and a fixed point (the focus "F"), formed by the points that are at the same distance from that reference line and from the parabola focus.**

We do not need the Cartesian graph to draw a parabola, as we see in Figure IV.5(a). We solely need its geometric law.

If we represent that parabola with the help of the Cartesian System[14], as in Figure IV.5(b), we may say that, in terms of algebra, the parabola is expressed by the equality "$y = k_2(x)^2 + k_1$"[15].

Figure IV.5: Parabola (geometric law and Cartesian representation)

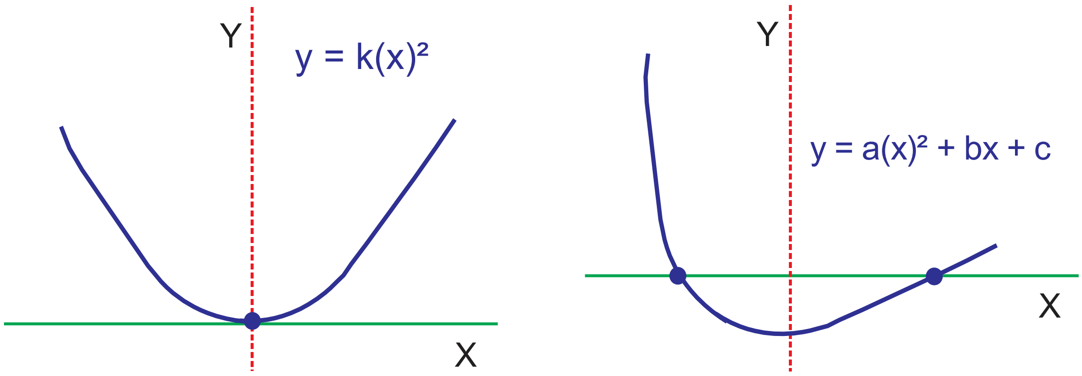


If we move the same parabola randomly in relation to the traditional Cartesian axes, we will find different equality expressions to describe that same parabola in terms of algebra. In Figure IV.6(a), the equality expression is "$y = k(x)^2$", while in Figure IV.6(b), the parabola is expressed by the equality "$y = ax^2 + bx + c$".

Keep in mind that we deal with a same geometric figure, a same parabola (geometrically defined), although expressed by an infinite number of math expressions in algebraic terms, with the employment of the traditional Cartesian System, which deals with positive and negative coordinates.

14 For clarity, when drawing the illustrations, I purposefully kept the straight line of reference and the orthogonal straight line passing through the focus as the Cartesian axes.
15 k_2" is the parameter that makes the geometric distinction between different parabolas.

Figure IV.6: Parabola (Cartesian representation)



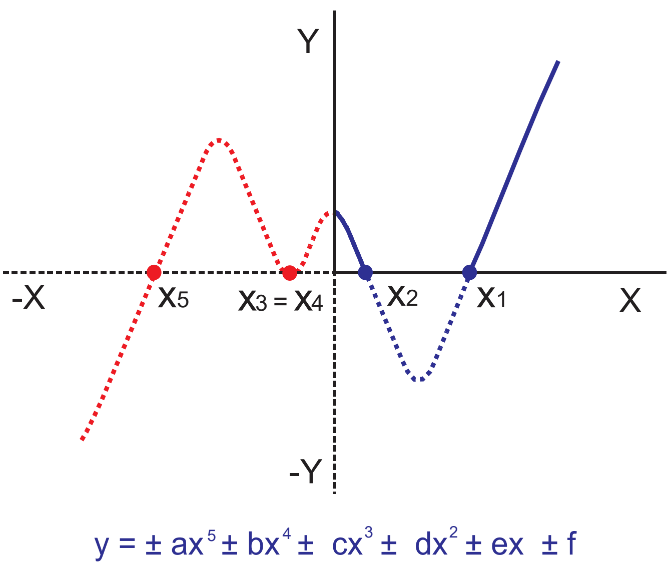
It is clear that (in representing a parabola) algebra interacts with geometry with the help of the Cartesian method. We see that the traditional Cartesian System imposes the use of coordinates with signs, but this is a consequence of the concept that numbers may have signs.

It is also clear that when we make "$y = 0$", and get the equality expression "$ax^2 + bx + c = 0$", we do not create an equation, and the resulting value for "x" (and "$y = 0$") is not a root of that so-called equation. The value of "x" when "$y = 0$", if any, simply means the abscissa of a point belonging to that parabola, which stays in the abscissa axis, for that specific position we placed a specific parabola in the Cartesian graph. We randomly placed a given parabola, geometrically defined, in the Cartesian graph.

As a second example, consider the 5th-degree polynomial represented in Figure IV.7.

According to the math concepts in force, when "$y = 0$" we create a 5th-degree equation, which must have five roots, either real or imaginary (when "x" is assumed as positive and as negative values). Please, keep in mind that a polynomial is a random algebraic expression, whatever its algebraic degree.

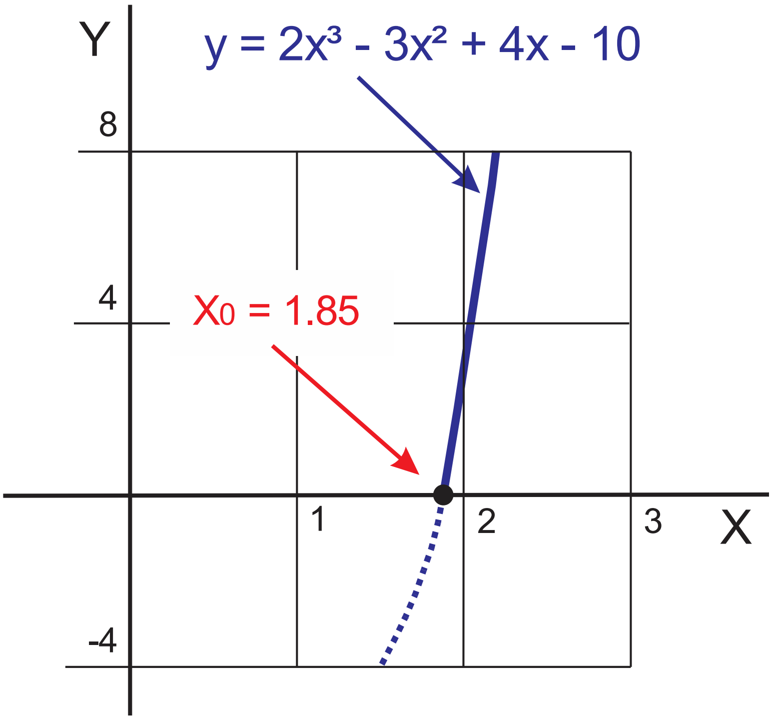
Figure IV.7: 5th-degree polynomial



As I will discuss at a later moment, we see in Figure IV.7 that the 5th-degree polynomial (and the geometric figure it represents) only exists in the 1st Cartesian quadrant (full line). Under certain assumptions (as the answer to a previously formulated question[16]), we may accept that the "y" value may be positive and negative, but "x" must remain as an absolute value (although a positive value under the traditional Cartesian System).

As a numerical example, consider the 3rd-degree polynomial "$y = 2x^3 – 3x^2 + 4x – 10$", as in Figure IV.8.

Figure IV.8: 3rd-degree polynomial



Again, according to the prevailing math concepts, if we make this 3rd-degree polynomial equal to zero ($y = 0$) we have a 3rd-degree equation, which has three real or imaginary roots, not represented in Figure IV.8 (when we assume "x" as a positive and negative values). In Figure IV.8, "$x_0 = 1.85$" is a root when "x" is assumed as a positive value.

Following the concepts of the New Math Model, these two polynomials do not represent typical geometric figures, and when we make them equal to zero ($y = 0$), they do not form equations and do not have any root. The abscissa value "$x_0 = 1.85$", when "$y = 0$", in the 3rd-degree polynomial only means that the difference between the aggregate amount of the terms subject to the plus (+) sign and the aggregate amount of the terms subject to the minus (–) sign is null[17] ("x" assumed as absolute value). Figure IV.8 clarifies this statement.

16 In case the reader is able to figure out a question like that (except ludic exercises), which requires the use of a 5th-degree polynomial.
17 There is no problem if math wants to keep calling that specific point as the "root" of the algebraic expression, provided the operator knows what he or she is doing, particularly the fact that he or she is not dealing with an equation.

In the first example, we chose a typical geometric figure (a parabola) and represented it in the Cartesian graph. We have a meaningful geometric figure that exists in nature, and obeys a geometric law, as well as a meaningful algebraic expression to represent it.

In the examples with polynomials, we chose random algebraic expressions (polynomials of different algebraic degrees) and plotted them in the Cartesian graph, and got geometric figures (flat curves), which do not exist in nature.

Then, we may pose a question:

> **Except as ludic exercises, what is the meaning and what are the applications of these random algebraic expressions (polynomials) and their corresponding geometric figures in our daily activities?**

CHAPTER V

FUNDAMENTAL AXIOM OF MATHEMATICS

Numbers and numbers hidden behind letters (either constants or variables) that form arithmetic and algebraic expressions are absolute values (modules). The plus (+) sign means addition operations, while the minus (-) sign means subtraction operations, only. That is the reason we do not need the so-called "rule of signs" to perform other arithmetic and algebraic operations.

Arithmetic and algebraic expressions

Math deals with different types of arithmetic and algebraic equalities, meaning math expressions formed with terms separated by the equality sign (=). These terms may be isolated numbers or letters, or a combination of numbers and letters. Moreover, the letters may work as constants or variables (the latter called "unknowns" when used in connection with equations).

In each side of the equality sign, we may have two classes of terms: the terms commanded by the plus (+) sign and the terms commanded by the minus (–) sign. It means we must, in each side of the equality, sum up the terms of each class (addition of absolute values), to get the balance, the difference between the total of the class commanded by the plus (+) sign and the total of the class commanded by the (–) sign. As a matter of fact, a subtraction between two absolute values[18].

We know that, by a universal convention accepted under the Traditional Math Model, we may omit the plus sign when placed in front of an isolated term or in front of the first term in any side of an equality expression. Nevertheless, we must keep in mind that these plus (+) signs are there, as emphasized in the following math expressions:

$$A = 2(3 + 1)$$

Actually means:

$$+ A = + 2(+ 3 + 1)$$

And,

$$B = 3(x – 2)$$

Actually means:

$$+ B = + 3(+ x – 2)$$

I will show that, under the New Math Model, that convention to ignore the plus (+) sign is not a valid approach, since it may cause the acceptance of improper

18 Obviously, when we transfer a term from one side to the other side of the equality, its commanding sign changes.

equalities.

We may omit, but not ignore the plus (+) sign in front of any term of a math equality.

Operations

A math equality is a sum of terms, either commanded by the plus (+) sign or by the minus (–) sign, as exemplified below:

$$+ y = + 2ax^3 - b(x^2 + 1) - 3cx$$

We also see that the terms that we will have to algebraically sum up may involve other operations, as multiplication, division, and others.

Current math understands that all these numbers and letters are positive or negative elements. If we assign positive and/or negative values to the constants and variables that form an algebraic expression, we may end up with odd and ambiguous arithmetic and algebraic operations, which require rules regarding operating priorities, as well as a rule of signs to perform multiplication, division, and other operations with positive and negative numbers and terms. I am convinced that students face great difficulty in understanding and dealing with this kind of unnecessary rules and conventions, it being a good reason to dislike the science of math.

I will demonstrate that, under the premises ruling the New Math Model, all these numbers and terms that form algebraic expressions are absolute values. We will treat arithmetic and algebraic expressions in a way to avoid the need of a rule of signs and of operating priorities, as well as the occurrence of those odd and ambiguous operations when dealing with math equalities. In other words, the plus (+) and minus (–) signs only mean addition and subtraction operations.

We must interpret the expression given above as follows:

$$+ |y| = + [|2|.|a|.|x^3|] - [|b|(|x^2| + |1|)] - [|3|.|c|.|x|]$$

In a true equality, we must find a final "y" result either of "$+ |y| = + |k|$" or "$- |y| = - |k|$". Otherwise, we would be dealing with a false equality ("$+ |y| = - |k|$" or "$- |y| = + |k|$").

My "Fundamental Axiom of Mathematics" is a "postulate", supported by logic reasoning and practical advantages, it being the reason I referred to it as an "Axiom". In fact, a proposition to reformulate some arithmetic and algebraic fundamentals.

It is relevant to remember that when using math to assist the other sciences, as in the determination of the geometric properties of geometric figures (length, area, and volume), and in the description of the behavior of physics phenomena (force, velocity, and others), we deal with absolute values of numbers and letters in the relevant scientific formula.

Before enunciating the Fundamental Axiom of Mathematics, it is also necessary to clarify the Author's concept of proper (true) and improper (false) math equalities, and introduce the special situations, under which math acceptably employs numbers with signs.

Proper and improper equalities

It is necessary to have in mind the Author's concept of proper (true) and improper (false) equalities, based upon the distinction between positive or negative values, and absolute values in math expressions, in order to understand his Axiom. In other words, the distinction between concepts within the algebra realm (including the representation of geometric figures in the Cartesian graph) and the real world.

Within current algebra, we deal with absolute values, as well as with positive and negative values, represented by positive and negative numbers or terms. In practice, said values represent things of the real world, which are neither positive nor negative (neutral things, represented by absolute values)[19]. That is why I postulate we must handle numbers and letters (constants or variables) in math expressions as absolute values; they are not positive nor negative values. We only use the plus (+) sign and the minus (–) sign to indicate addition and subtraction operations with numbers and letters, as the terms (absolute values) of math equalities.

The concept of proper and improper equalities is an approach I will use in connection with the New Math Model here proposed. To my knowledge, a highly subtle concept that the Traditional Math Model does not contemplate.

It is my understanding that present math concepts work within the 1st and the 3rd quadrants ("x" and "y" with equal signs), but, to avoid the use of the 3rd quadrant, algebra mistakenly accepts positive numbers as equivalent to absolute values, and the existence of complex numbers.

In an equality, we split elements of a same nature (as dollars) into two classes (as credits and debts). These elements are absolute values. As a convention, we use the plus (+) sign in front of each element of one class (as credits), and the minus (–) sign in front of each element of the other class (as debts), whatever the side of the equality each term is placed. Obviously, if we move a term of an equality from one side to the other side of the equality, we must change its commanding sign.

To find the difference between the total values of the two classes (balance), we perform the "algebraic sum", which in fact is a "subtraction operation" of the sum of

19 I mean things of day-to-day use, as assets and goods (dollars, cars), properties of geometric figures (areas, volumes), elements of physics (weights, velocities) and the like, which do not have signs.

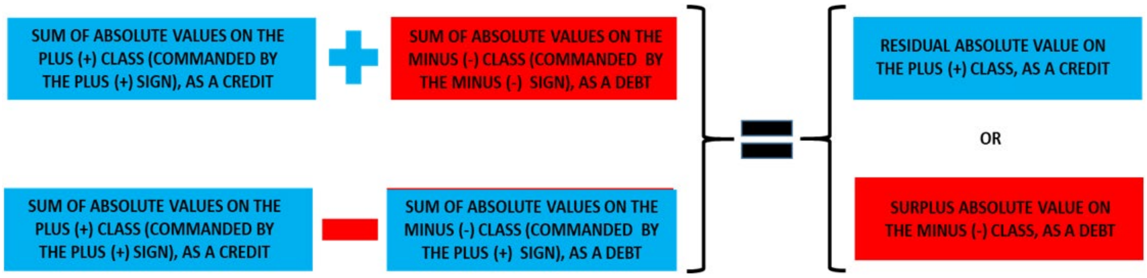
the absolute values of one class from the sum of the absolute values of the other class, as illustrated below:

Balance = (+ $|m_1|$ + $|m_2|$ + …) plus (– $|n_1|$ – $|n_2|$ –…) → (algebraic sum)

Balance = (+ $|m_1|$ + $|m_2|$ + …) minus (+ $|n_1|$ + $|n_2|$ +…) → (subtraction)

Figure V.1 shows the concepts of algebraic sum (addition and subtraction), as well as the resulting balance, either a positive residual or a negative surplus (if not zero).

Figure V.1: Algebraic sum (addition and subtraction)



We use the same addition operation to obtain the two class totals: the sum of the terms (absolute values) commanded by the plus (+) sign, and the sum of the terms (absolute values) commanded by the minus (–) sign. Then, we perform the algebraic sum of the two classes to get the balance, the latter, in fact, a subtraction operation.

Obviously, any of the two classes may prevail in the result (balance), but in all these operations, we deal with absolute values (either isolated numbers or terms formed with numbers and letters).

To clarify the concept, let us find the unknown account balance “z” of a bank client by knowing that he or she has a total amount of credits equal to “x” (x = 100 dollars) and a total amount of debts equal to “y” (y = 120 dollars). As usual, let us further assume that credits are positive figures, while debts are negative figures (clearly, a convention, since currencies, as dollars, are neutral values, neither positive nor negative elements).

We then may write:

$$z = x - y$$

Algebraically speaking, we mistakenly (Traditional Math Model) think of:

$$+ z = + x - y$$

And,

$$+ z = + 100 - 120 = - 20$$

$$+ z = - 20$$

Then, algebra tells us that said person has a credit (a positive amount "+z") equal to a debt (a negative amount "– 20"), a clearly mistaken response (an improper equality).

In fact, we should understand that problem (New Math Model) as:

$$|z| = |x| - |y|$$

Following the numerical example:

$$|z| = |100| - |120| = -|20|$$

$$|z| = -|20|,$$

This result has a clear interpretation, since it allows us to conclude that the person's account balance (initially unknown "|z|") is a debt (– 20). We assumed an absolute value to represent the unknown accounting balance "z" of that bank client, since said balance may be a credit or a debt (a positive or a negative amount by convention).

That is the reason to avoid the confusion between positive values and absolutes values. We cannot ignore the plus (+) sign in any term of an equality, as it happens when we deal with the Traditional Math Model. We need to consider the existence of the plus (+) sign in front of all terms, which belong to the class considered as the positive class, no matter their position in the algebraic equality (as an isolated term or the first term in any side of the equality).

As we will see, we face this dilemma because within the algebra realm we deal with positive and negative values, as well as with absolute values. However, the Cartesian method does not contemplate absolute values, unless we remain in the 1st or in the 3rd quadrant of the graph (both coordinates with a same sign). In the reality realm, we only have neutral values.

Field of validity of an equality

Consider the algebraic equality "$y = \sqrt{(5 - x)}$". We actually mean that "$+ y = + \sqrt{(+ 5 - x)}$". Whenever the "x" value is less than or equal to "5", we will have a square root of zero or of a value commanded by the plus (+) sign (let us say "$\sqrt{+ a}$" or "$+ \sqrt{a}$") on the right side of the equality. Then we will end up with an equality "$+ y = + \sqrt{a}$", a valid operation under the concepts of the Traditional Math Model.

In case "x" is greater than "5", we will face the square root of a value subject to the minus (–) sign (let us say "$+ \sqrt{- a}$"), and under the rules of the Traditional Math Model we enter the realm of imaginary numbers, "$+ y = + \sqrt{- a}$".

Under the concepts of the New Math Model, here introduced, we have a different interpretation:

(i) the expression “$+ y = + \sqrt{(+ 5 – x)}$”, similarly to what happens with the Traditional Math Model, “y” only exists for “x” less than or equal to “5”, which yields the result “$+ y = + \sqrt{+ a} = + \sqrt{a}$”. Then, “x” less than or equal to “5”, is the “existence field” of “$+ y$”; and

(ii) the expression “$– y = + \sqrt{(+ 5 – x)}$”, “y” only exists for “x” equal to or greater than “5”, which yields the result “$– |y| = + \sqrt{– |a|}$” $= – \sqrt{|a|}$”. Then, “x” equal to or greater than “5”, is the “existence field” of “$– y$”.

In both situations above, items “i” and “ii”, we face proper equalities, since the absolute values, “y” and “its result”, show the same sign. In other words, we are working within the 1st and the 3rd Cartesian quadrants, where the Cartesian coordinates of “y” and “x” have equal signs. It is relevant to emphasize that it does not matter if we are dealing with a power or a rooting, nor if the power exponent or the rooting index is an even or an odd number.

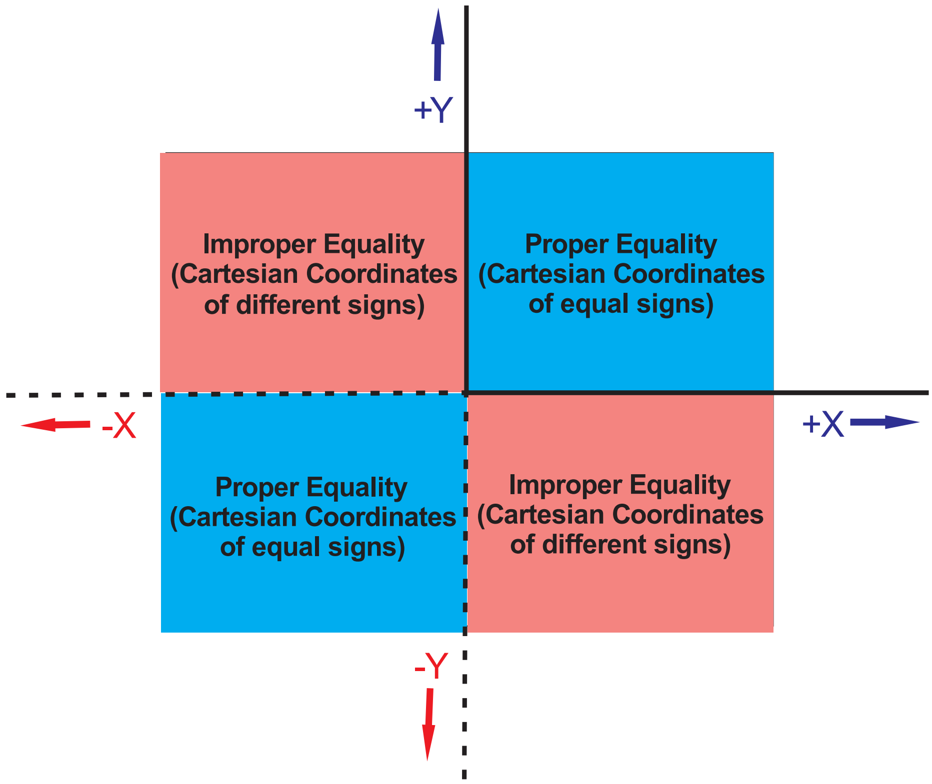
In the same way we cannot accept the math expression “$+ y = + \sqrt{– a}$”, we cannot accept the math expression “$– y = + \sqrt{+ a}$”, because we will be dealing with improper equalities. In other words, we will be working within the 2nd and the 4th Cartesian quadrants, where the Cartesian coordinates of “y” and “x” have different signs.

That is why we do not need the theory of complex numbers. We are not dealing with imaginary numbers, but with improper equalities. The theory of complex numbers is a misuse of algebra under the Traditional Math Model.

It is relevant to clarify that, under the premises of the New Math Model, the validity of a math expression is not related to the (even or odd) rooting index in rooting operations, but dependent on the concept of proper and improper equalities. In the expressions “$+ y = (5 – x)$” and “$– y = (5 – x)$”, the variable “y” has the same condition of existence as in the expressions “$+ y = \sqrt{(5 – x)}$” and “$– y = \sqrt{(5 – x)}$” above discussed (“x” less than or equal to “5” and “x” equal to or greater than “5”, respectively).

Figure V.2 illustrates the concept of algebraic proper and improper equalities in connection with the Cartesian System.

Figure V.2: Cartesian view of proper and improper equalities



Rule of signs

I accepted the concept that numbers and terms forming equalities expressions are absolute values. We saw that we use the plus (+) sign to mean addition of elements on the positive class, as well as of elements on the negative class of the equality. Moreover, we also use the same plus (+) sign to mean the algebraic sum[20], which actually is a subtraction, an operation indicated by the minus (–) sign.

This concept has effects on the multiplication, division, and other operations with numbers and terms. Traditionally, these operations deal with positive and negative numbers and terms. This understanding requires the so-called "rule of signs", under which a multiplication or a division operation between numbers and/or terms with the same sign yields a positive result, while a multiplication or a division operation between numbers and/or terms with different signs yields a negative result, whatever the order of the elements in the operation.

I will show that these operational effects are correct, but the explanation is different. There is no rule of signs applicable to multiplication and division operations with positive and negative elements, but solely the application of addition and subtraction rules to multiplication and division between absolute values, as we usually do.

To explain the reasoning, let us consider multiplication and division operations:

In case, for any reason, we face an operation of the type "$\pm z = (\pm x)(\pm y)$", as above,

20 Two different meanings for the same symbol, the plus (+) sign.

we must keep in mind that we are dealing with absolute values (modules) of numbers or terms. Actually, we must understand the operation as "$\pm |z| = (\pm |x|)(\pm |y|)$". Then:

$$\pm |z| = (-|x|)(-|y|) = -(|x|)(-|y|) = +(|x|)(|y|) = +xy$$

$$\pm |z| = (-|x|)(+|y|) = -(|x|)(+|y|) = -(|x|)(|y|) = -xy$$

We simply follow addition (+) and subtraction (–) commandments. The plus sign (+) before an absolute value tells us to keep the sign before the other absolute value (whatever said other sign); similarly, the minus sign (–) before an absolute value tells us to change the sign of the other absolute value (also, whatever said other sign), as we always do in relation to addition and subtraction operations.

The so-called "rule of signs" does not exist, except as a rule of thumb (a mnemonic help). We deal with numbers (or terms) which are neutral elements, since the plus (+) sign and the minus (–) sign before a number (or a term) only mean addition and subtraction operations.

Similarly, in case we want to perform the operation below:

$$z = (x - a)(-x - b)$$

Under the Traditional Math Model, we mistakenly think of:

$$+z = (+x - a)(-x - b)$$

While, under the New Math Model, we should think of:

$$+|z| = (+|x| - |a|)(-|x| - |b|)$$

And

$$+|z| = -|x|^2 + (|a|)(|x|) - (|b|)(|x|) + (|a|)(|b|)$$

The plus (+) and minus (–) signs solely mean addition and subtraction operations. In the example above, I simply applied the traditional addition and subtraction commandments.

If we consider powers and roots, we may reach the same conclusions, as follows[21]:

$$(-3)^2 = -(|3|^2) = -(|9|) = -9$$

Similarly,

$$\sqrt{(-9)} = \sqrt{(-|9|)} = -\sqrt{|9|} = -3$$

This approach is entirely different from the current approach. We will apply the same operating rules to numbers and terms, whatever the sign in front of them, since we will consider all of these numbers and terms as absolute values.

21 It is relevant to mention that such type of operations will only occur in connection with math expressions, not as isolated terms. They have no meaning or use as isolated terms.

In multiplication and division operations, the plus sign (+) before the first element tells us to keep the sign of the other element in the result. That is why "plus (+) and plus (+) yield plus (+)", and "plus (+) and minus (–) yield minus (–)": (+ a)(+ b) = + (ab), and (+ a)(– b) = – (ab).

Contrarily, the minus sign (–) before the first element tells us to change the sign of the other element in the result. That is why "minus (–) and minus (–) yield plus (+)", and "minus (–) and plus (+) yield minus (–)": (– a)(– b) = + (ab), and (– a)(+ b) = – (ab).

In case of a power operation, the sign in front of the power tells us the power result must have the same sign of the power in case of the plus (+) sign), or change it in case of the minus (–) sign. Then, whatever the power exponent: $+ (+ a)^n = + (a^n)$, $+ (– a)^n = – (a^n)$, $– (+a)^n = – (a)^n$, and $– (–a)^n = + (a)^n$.

In case of a root operation, the plus (+) sign in front of the root symbol tells us the root must have the same sign inside the root symbol, while the minus (–) sign in front of the root symbol tells us the root must have a sign different from the sign inside the root symbol. Then, whatever the root index: $+ (\sqrt[n]{+ a}) = + (\sqrt[n]{a})$, $+ (\sqrt[n]{– a}) = – (\sqrt[n]{a})$, $– (\sqrt[n]{+ a}) = – (\sqrt[n]{a})$, and $– (\sqrt[n]{– a}) = + (\sqrt[n]{a})$.

The operations above discussed will only appear when we deal with math equalities, because addition and subtraction of terms are the sole operating commandments of the plus (+) and minus (–) signs in any math expression (not applicable to isolated terms, as seen above).

Numerical examples will emphasize this concept that we only associate the plus (+) and the minus (–) signs to numbers and letters when they are terms of math expressions:

$\ldots + (– 3^2) = \ldots + (– |3^2|) = \ldots – |3^2| = \ldots – 9$

$\ldots – (– 3^2) = \ldots – (– |3^2|) = \ldots + |3^2| = \ldots + 9$

As previously stated, that is the reason we do not need the so-called rule of signs and the theory of complex numbers.

I will show other numerical examples in Appendix "A".

Valid use of numbers and letters with signs

There are three situations, under which numbers may acceptably appear as positive or negative elements in connection with math matters. However, these special situations do not contradict the Fundamental Axiom of Mathematics.

(i) **When dealing with true equations[22], a case in which we face a forced algebraic equality of a single unknown, and search for the value of the unknown that satisfies the given algebraic equality. As per the Axiom, we assume the expected value of the unknown as being an absolute value, meaning we are dealing with a proper equality, what may not be true. The resulting value for the unknown may be positive, negative or null, since said unknown value must have a meaning, as the answer to a previously made question (that is the rationale behind an equation).**

(ii) **By math conventions, as:**

(a) The use of a negative exponent to handle a power used as the denominator of a fraction ($1/x^n = x^{-n}$), only to facilitate the operations. In derivatives, for instance, if we consider the math expression "$y = x^{-2}$" and find its derivative (following the applicable rules), we see:

$y' = -2x^{-3}$ or $y' = -(2/x^3)$

However, if we apply the derivative rules to the math expression "$y = 1/x^2$", we will get the same result:

$y' = [(0)(x^2) - (1)(2x)]/x^4 = -(2/x^3)$

Apparently, we are dealing with negative numbers, since the math expression exhibits a negative exponent. In fact, the math expression "$y = x^{-2}$" has no derivative because it does not exist, except as a convention, a different notation for a true math expression "$y = 1/x^2$". There is no negative number involved in the given math expression.

b) The representation of a geometric figure under the prevailing rules applicable to the Cartesian System, which requires positive and negative coordinates, knowing that we are performing unnecessary ludic exercises. Even when we are dealing with the traditional Cartesian System, only the 1st quadrant of the Cartesian graph has relevance.

Fundamental Axiom of Mathematics

Then, we may finally enunciate the Fundamental Axiom of Mathematics:

Numbers and letters, either constants or variables, as well as their resulting terms, which form valid arithmetic and algebraic math expressions (proper equalities), do not have signs; they are absolute

22 A true equation is an equality between two algebraic sums, which have a single unknown value. The resulting unknown value may be positive, negative or null, since it has to be the answer to a previously formulated question.

values (modules).

Corollary 1:

Numbers and letters commanded by signs, either the plus (+) sign or the minus (-) sign, only appear as terms of math expressions. They do not have meaning or use as isolated terms.

Corollary 2:

Whatever the operation we perform, the plus (+) sign and the minus (–) sign in front of numbers and letters solely mean addition and subtraction operations.

CHAPTER VI

ALTERNATIVE CARTESIAN SYSTEM

The concept that numbers are neither positive nor negative elements allowed me to enunciate the Fundamental Axiom of Mathematics. This Axiom will take me to the Alternative Cartesian System. The next unavoidable step will be a New Math Model.

Concept of Cartesian coordinates

Under current math concepts, we use positive and negative coordinates in connection with the Cartesian System. This understanding is a natural inference from the prevailing concept that numbers are positive or negative elements.

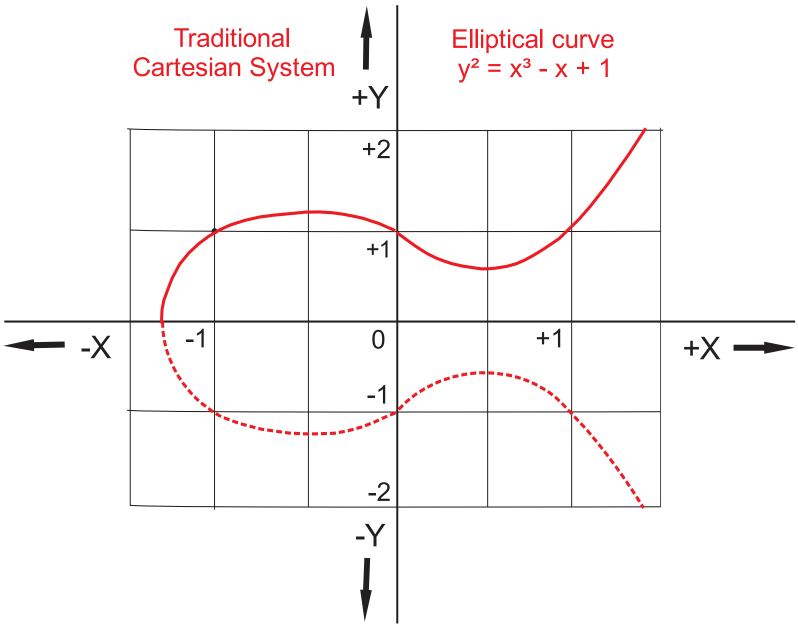
It means that in any algebraic equality "y = f(x)", the variables, "x" and "y", must assume positive or negative values. As a result, the equality may have all four combinations of results for the values of the variables, as ("+ x and + y", "+ x and – y", "– x and + y", and "– x and – y"). It also means that algebra accepts the use of improper algebraic equalities.

This approach requires the need of the four quadrants of the Cartesian graph to represent the algebraic expression, no matter if we deal with a typical or with a modified geometric figure.

We see this situation in Figure VI.1, when we used the well-known elliptical curve as an example.

$+ y^2 = + x^3 – x + 1$.

Figure VI.1: Elliptical curve according to the traditional Cartesian System



The symmetry observed in Figure VI.1, in relation to the abscissas axis, results from the fact that, under the premises of the Traditional Math Model (rule of signs), when we deal with a relationship as "$y^2 = f(x)$", for any value "$y^2 = k$", "$y = \pm \sqrt{k}$".

We also recall that, by a universal convention, we may omit the plus (+) sign in relation to isolated terms, as "$+ y^2$", and "$+ x^3$", but I used them in the algebraic expression to remind the reader that these signs do exist, and cannot be ignored.

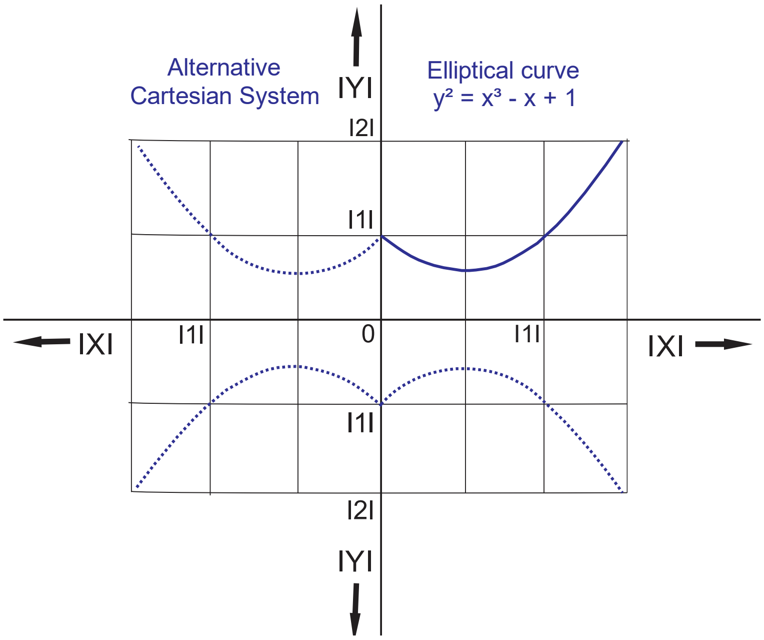
Under the New Math Model, the variables "x" and "y" are neutral values in any algebraic equality. As a result, a proper algebraic equality may only have two combinations, (+ x and + y), and (– x and – y).

This alternative approach only requires a single quadrant of the Cartesian graph to represent the algebraic expression we consider, it being a typical or a modified geometric figure.

We see this situation in Figure VI.2, when we used the same well-known elliptical curve as an example. According the rules of the New Math Model, we must interpret the algebraic expression of that given elliptical curve as follows:

$$+ |y|^2 = + |x|^3 - |x| + |1|$$

Figure VI.2: Elliptical curve according to the Alternative Cartesian System



The curves are the same in all four Cartesian quadrants. Again, I maintained the plus (+) signs in the terms "$+ y^2$", and "$+ x^3$" as a reminder that these signs do exist.

As in the example with the elliptical curve, in my propositions I adopted an opposite understandings: numbers and letters in math expressions and the coordinates in the Cartesian System are absolute values (modules). I refer to this approach that Cartesian coordinates are modules as "Alternative Cartesian System", an extension of the concept stated under the Fundamental Axiom of Mathematics.

The Alternative Cartesian System requires a single quadrant to represent any geometric figure in algebraic terms, but it is possible to use the four quadrants (symmetrical geometric figures), as the traditional Cartesian System does. In using the four quadrants, all semi-axes start from a common origin "O", and in spite of following opposite directions, the coordinates are neither positive nor negative figures, but modules. The Cartesian System is a method that algebra uses to represent geometric figures in terms of algebra or an algebraic expression as a geometric figure, a math tool to serve algebra and geometry.

In the example, we treated a same algebraic expression ($y^2 = x^3 - x + 1$), known as elliptical curve, under two different concepts:

In Figure VI.1 under the premises applicable to the Traditional Math Model, which state that numbers and letters in algebraic expressions, and coordinates of the Cartesian System are positive or negative elements, what requires the use of the four Cartesian quadrants.

In Figure VI.2, under the New Math Model, which states that numbers and letters in algebraic expressions, and coordinates of the Cartesian System are absolute values (modules), and we need a unique Cartesian quadrant.

Geometrically speaking, we see two entirely distinct curves as the Cartesian representations of a same algebraic expression. It is worthwhile to remember that an elliptical curve is a modified geometric figure, which does not exist in nature. Since it is not a typical geometric figure, and does not exist in nature (and contrarily to other common typical geometric figures), nobody can tell what is the real shape of an elliptical curve, even knowing its algebraic expression.

As a side comment, I know that it is possible to use elliptical curves in cryptography and other math applications. This is a subject outside my field of knowledge and interest, but my innovative propositions will not prevent said practical use of elliptical curves.

New concept of Cartesian coordinates

With the purpose to clarify the new concept of Cartesian coordinates, let us analyze the traditional geometric figure of a parabola. The algebraic expression of a parabola, in its simplest algebraic form, with its focus on the vertical axis (later on, the ordinate axis in the Cartesian graph), is as follows:

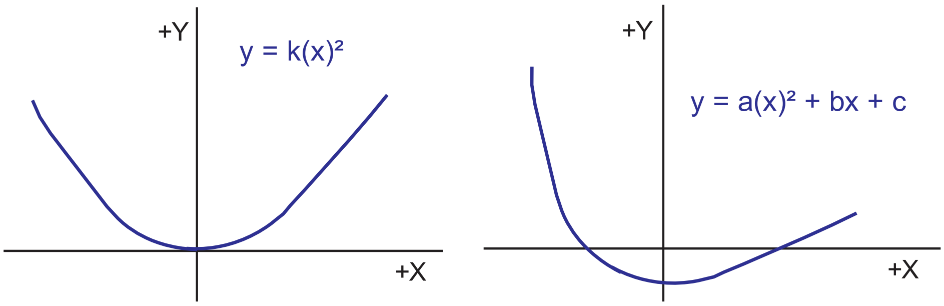
$$y = kx^2$$

We also may express the same parabola (geometrically defined), by an infinite number of algebraic expressions, as follows:

$$y = ax^2 + bx + c$$

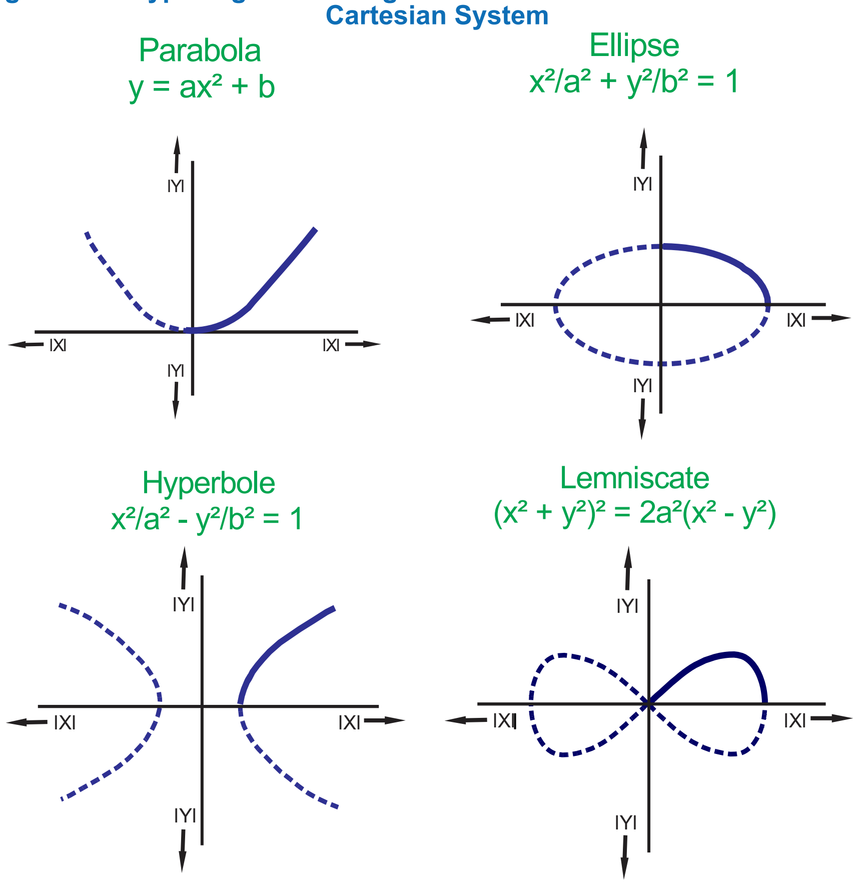
Figure VI.3 illustrates this understanding, a same parabola in two different positions in the Cartesian graph, and their respective algebraic expressions, in accordance with the traditional Cartesian System.

Figure VI.3: Representations of a Parabola (traditional Cartesian System)



Following my suggestion of an Alternative Cartesian System, we have to use the simplest position to represent a typical geometric figure[23], geometrically defined, in the Cartesian graph. Once placed in its simplest position, we express the geometric figure by the simplest algebraic expression, as in Figure VI.4. In case of a single symmetry axis (as a parabola), we also may displace the geometric figure vertically or horizontally in relation to the symmetry axis. When we deal with geometric figures with two or three symmetry axes, there is a unique position for the geometric figure (as an ellipse and an ellipsoid).

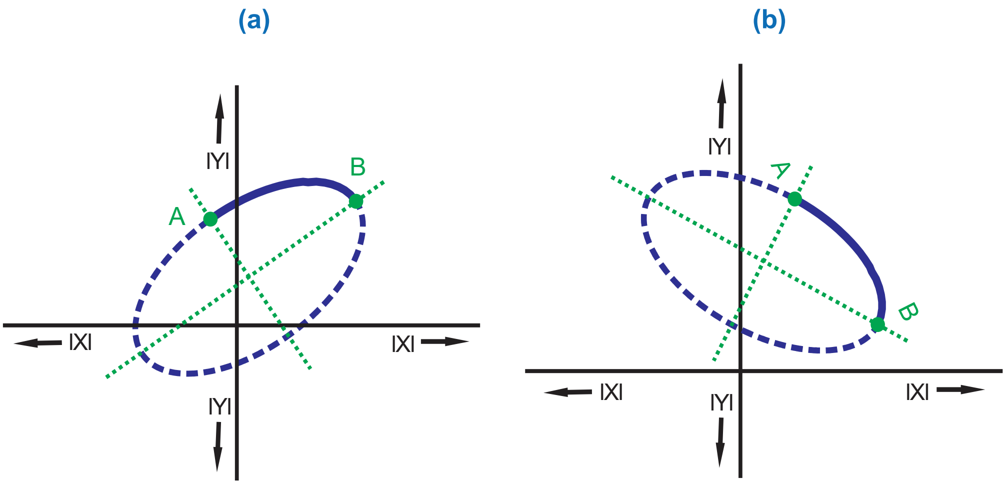
Figure VI.4: Typical geometric figures in accordance with the Alternative Cartesian System



23 See concept in Chapter IV of this book (typical and modified geometric figures).

In using the Alternative Cartesian System, it is not convenient to place a geometric figure in different positions, other than the simplest one, as in Figure VI.5.

Figure VI.5: Ellipse in a different position



The option for this type of more complex algebraic expression is exactly the approach I say we must avoid by adopting the Alternative Cartesian System. The representative portion of the geometric figure (as the curved segment “AB” in the above ellipse) may reach more than one Cartesian quadrant, as in Figure VI.5(a). Even if technically possible, as in Figure VI.5(b), what is the practical reason to use such unnecessary approach?

If a person wants to perform this type of dilettante exercises, he or she may use the traditional Cartesian System, with positive and negative coordinates, but knowing he or she is performing a ludic exercise.

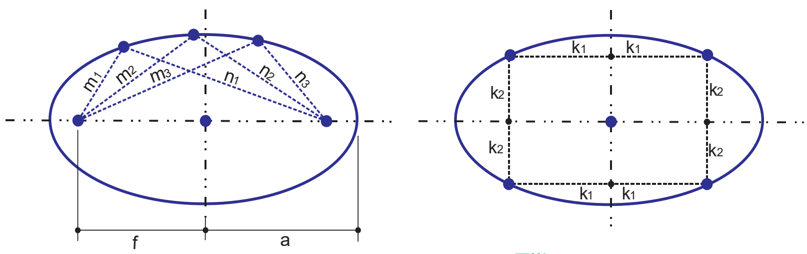
> **With the use of the Alternative Cartesian System, we only need a single quadrant and the coordinates as absolute values.**

Demonstration of the Fundamental Axiom of Mathematics

It is possible to demonstrate my Axiom previously referred with the help of geometry. Figure VI.6(a) shows the geometric law that defines the ellipse: “The flat figure formed by the points, for which the sum of their distances to two fixed points is constant.”

Figure VI.6: Geometric law and Cartesian representation of an ellipse

(a) (b)



Ellipse: geometric law
$m_1 + n_1 = m_2 + n_2 = m_3 + n_3 = 2a$

Ellipse: symmetry

In math terms,

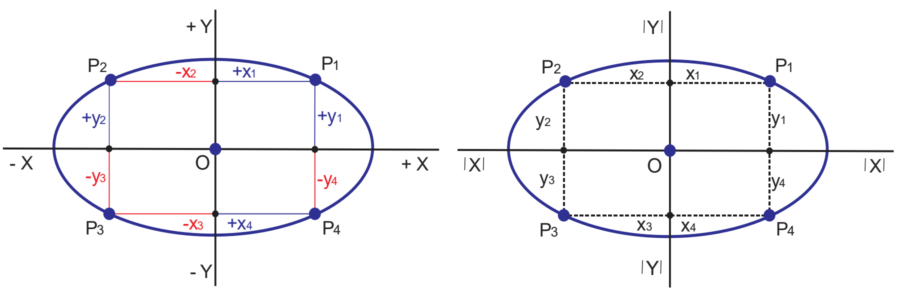
$$m_1 + n_1 = m_2 + n_2 = m_3 + n_3 = 2a$$

Figure VI.6(b) indicates that all these values are absolute values.

Figure VI.7(a) shows the same ellipse represented in accordance with the present understanding of the Cartesian System, where we see positive and negative signs. In fact, we placed the same absolute values, in the same position, but obeying the Cartesian convention of positive and negative coordinates. In Figure VI.7(b) we see the same ellipse in the Alternative Cartesian System, in which all coordinates are absolute values, starting from a common origin “O”.

Figure VI.7: Analysis of an ellipse

(a) (b)



Ellipse: coordinates (with signs)

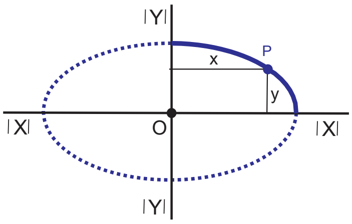
$|x_1| = |x_2| = |x_3| = |x_4| \quad |y_1| = |y_2| = |y_3| = |y_4|$

Ellipse: coordinates (absolute values)

$|x_1| = |x_2| = |x_3| = |x_4| \quad |y_1| = |y_2| = |y_3| = |y_4|$

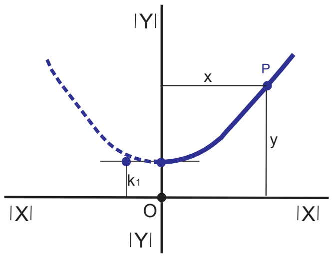
Figure VI.8 indicates it is not necessary to work with positive and negative coordinates.

Figure VI.8: Use of the Alternative Cartesian System



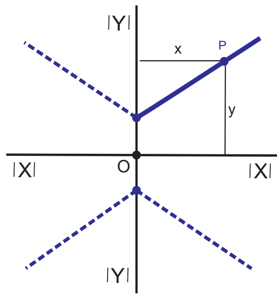
Ellipse: formula (absolute values)

$$\frac{|x^2|}{|a^2|} + \frac{|y^2|}{|b^2|} = |1|$$



Parabola: formula (absolute values)

$$|y| = |k_2||(x)^2| + |k_1|$$



Straight line: formula (absolute values)

$$|y| = |a||(x)| + |b|$$

We see that, in accordance with the Alternative Cartesian System, the math expression of any geometric figure (with absolute values) works well in any of the four Cartesian quadrants, whatever the geometric figure, ellipse, parabola, straight line and other geometric figures.

As seen in the illustrations, I omitted (but I did not ignore) the plus (+) sign in front of certain terms of the math expressions, since all of them are absolute values.

As previously stated, it is relevant to clarify that my Axiom does not prevent the use of numbers with signs, provided the operator knows what he or she is doing. The value of the unknown variable in a true equation, the coordinates of geometric figures in a Cartesian graph (although unnecessary), or the convention to use negative exponents in the numerators to represent exponent in fraction denominators are examples of valid uses of positive and negative signs in front of numbers in math applications.

Any person may also perform ludic exercises with the traditional Cartesian System, by using positive and negative coordinates. However, it is mandatory that he or she be aware of what he or she is doing.

CHAPTER VII

MATH MODELS

Math relies on certain fundamentals, as the concept of numbers, which yield the math model in use. If we change any of these fundamentals (as the concept of numbers), we may end up with a significantly different math model.

Numbers and letters in algebraic expressions

Contrarily to the meaning assigned to numbers and letters in math expressions, and to the coordinates of the Cartesian graph under the concepts of the Traditional Math Model, I started my reasoning by adopting the concept that isolated numbers are neutral elements. Following this concept, I enunciated the Fundamental Axiom of Mathematics, which extends the same interpretation to letters in any algebraic expression. The Axiom pushed me towards the Alternative Cartesian System, with coordinates as absolute values. The next unavoidable step is a New Math Model, certainly different from the Traditional Math Model in force.

After introducing the concepts applicable to the New Math Model, by presenting new understanding about the nature of numbers and letters in math expressions, and the use of absolute values as the coordinates in connection with the Cartesian System, let us compare the three previously referred math structures (math models):

(i) The Traditional Math Model in use;

(ii) The first alternative math structure, under the speculative assumption that the same operating rules apply to positive and to negative numbers; and

(iii) The New Math Model proposed in this book.

To illustrate the differences, consider the traditional parabola, a well-known geometric figure, which we often express in the algebraic Cartesian form by:

$$y = ax^2 + bx + c$$

Traditional Math Model

According to the concepts applicable to the Traditional Math Model, if we make the parabola Cartesian expression equal to zero ($y = 0$), "$ax^2 + bx + c$" $= 0$", we have the so-called 2nd-degree equation, which may show real or imaginary roots, as listed below:

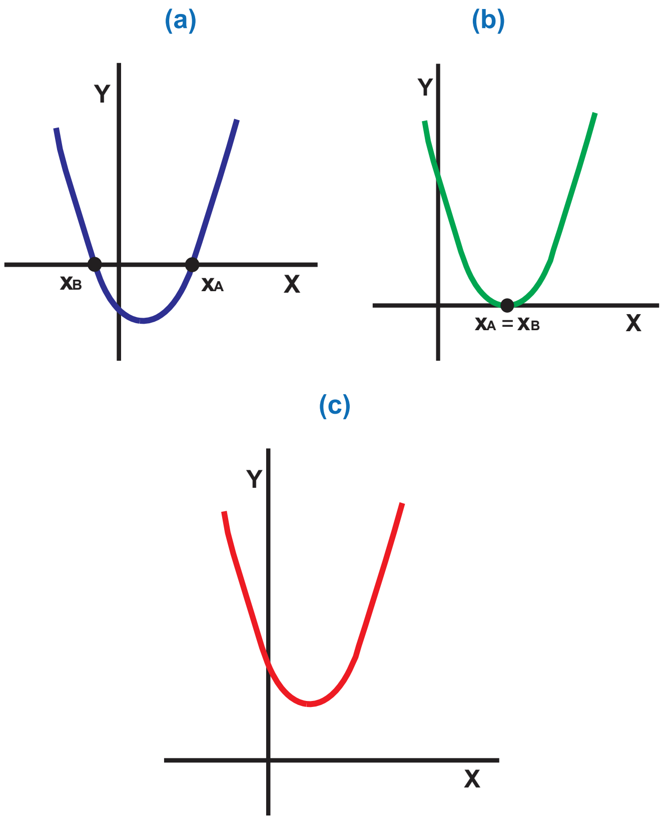
(i) Two different real roots (x_A and x_B).

(ii) **One real root or two equal real roots ($x_A = x_B$).**

(iii) **No real root.**

Figure VII.1 illustrates the geometric figure of a parabola as seen in these different Cartesian positions.

Figure VII.1: Cartesian representation of a parabola



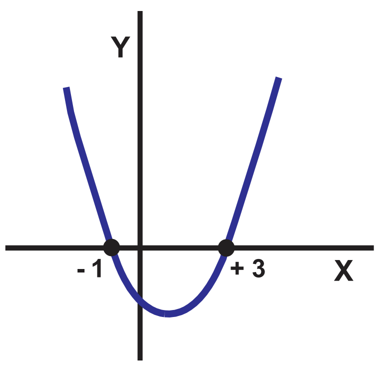
In the numerical example, the parabola "$y = x^2 – 2x – 3$" yielded the 2nd-degree equation "$x^2 – 2x – 3 = 0$". As we see in Figure VII.2, according to the Traditional Math Model, that 2nd-degree equation has the following two roots:

$x_A = + 3$

$x_B = – 1$

It is obvious that under the number assumption of the Traditional Math Model we find two real roots, and both roots satisfy the original expression, making "$y = 0$".

Figure VII.2: Roots of a 2nd degree equation (Traditional Math Model)



First alternative math structure

I do not intend to spend any effort on this conceptual approach, because I believe the development of said math structure will lead us to a highly complicate operating procedure. Said that, in order to analyze the results of this possible math model, let us accept a speculative and questionable assumption that the rules applicable to positive numbers also apply to negative numbers. To my knowledge that math structure (and said rule of signs) does not exist.

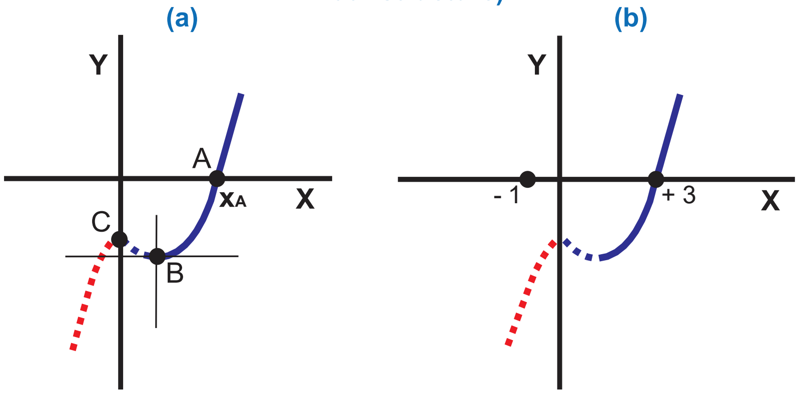
Following the rules applicable to the Traditional Math Model, we got "$x_A = +3$" and "$x_B = -1$" as the roots of the math expression "$x^2 - 2x - 3 = 0$".

If we use the rules of the first alternative math structure under the speculative and questionable assumption made above (whatever the meaning and consequences of said rule of signs) to test the roots found, we see that:

> **$x_A = +3$, yields "$y = 0$", and the root satisfies the supposed equation; but**
>
> **$x_B = -1$, yields "$y \neq 0$", and this root does not satisfy the supposed equation.**

We then find a single root ($x_A = +3$) for the so-called 2nd-degree equation, "$x^2 - 2x - 3 = 0$", as illustrated in Figures VII.3(a) and VII.3(b). Under said assumption, no imaginary root would exist.

Figure VII.3: Parabola and 2nd degree equation (under the first alternative math structure)



Besides the possible difficulties with the operational treatment we should adopt to handle positive and negative numbers (regarding the need of a consistent rule of signs), it is my impression that working with the "first alternative math structure" would be even more complicated than with the Traditional Math Model. Good reasons not to move further with said approach.

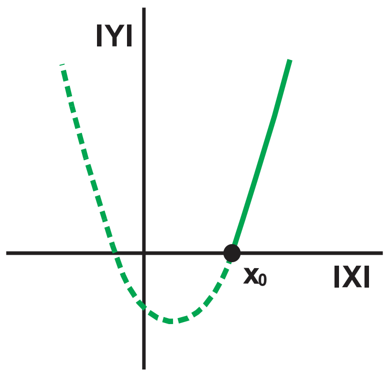
New Math Model

In previous comments, I proposed different interpretations and uses of some basic math premises presently in force. These alternative interpretations and uses of certain math fundamentals lead us to a New Math Model (also, as far as I know, a non-existing math model).

As we already discussed, according to the concepts of the New Math Model, the negative value of "x" obtained with the Traditional Math Model ($+ x_B = -1$) only means that we are dealing with an improper (false) equality[24].

It is obvious that it is possible to find a value for "x" in the algebraic expression, "$\pm ax^2 \pm bx \pm c = 0$", which (if exists) only means the abscissa value where the original parabola, "$\pm y = \pm ax^2 \pm bx \pm c$" crosses or touch the abscissa axis ($x = x_0$), as we see in Figure VII.4. A point (or points) in the parabola curve, where the ordinate "$y = 0$" for a particular position of that parabola in the Cartesian graph. Under the New Math Model, the algebraic expression, "$\pm ax^2 \pm bx \pm c = 0$", is not an equation, unless there is a meaning behind the "x" value, other than the abscissa of a point belonging to that parabola.

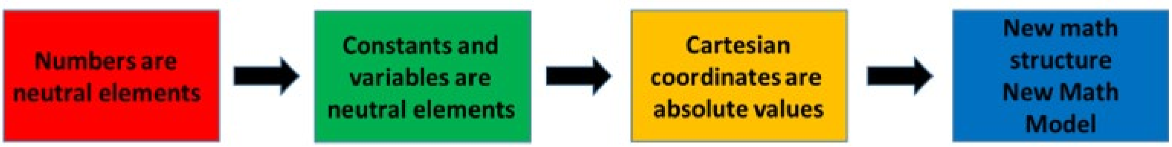
Figure VII.4: Parabola crossing the abscissa axis



In summary, the concept that numbers are neutral elements led me to the Fundamental Axiom of Mathematics, by extending that concept of numbers to letters and terms in any algebraic expression. The Axiom suggested me the Alternative Cartesian System, the use of absolute values as the coordinates of the System. The next unavoidable step was the acceptance of a New Math Model, a different math structure consistent with the new math premises proposed in this book.

I illustrated this reasoning sequence in Figure VII.5.

Figure VII.5: Sequence of new concepts



In Chapter VIII, I will make a comparative analysis about the different understandings of some basic math concepts under the Traditional Math Model and the New Math Model. In Appendix "A", I will test my innovative concepts by dealing with some elementary math exercises under the premises of these two math models.

24 In fact, under the New Math Model, the given algebraic expression is not an equation and has no root.

CHAPTER VIII

BASIC CONCEPTS UNDER THE TWO MATH MODELS

To understand the New Math Model, we need to see how it differs from the Traditional Math Model in what concerns the interpretation and use of some math fundamentals.

Summary of concepts

The basic difference between the Traditional Math Model in force and the New Math Model here proposed is the meaning each model attributes to numbers. Then, other subsequent differences appear, as discussed in this book. These different conceptual approaches, as accepted by each model, will lead to distinct math structures, generating different math models.

Notwithstanding that, the two math models yield the same "useful results", as needed by common people and scientists in their daily endeavors. This fact raises a question:

> **Why should anyone care to adopt a new math structure, which will provide the same "useful results" provided by the current math structure?**

That is the point to analyze. In addition to the "useful results", the Traditional Math Model coexists with odd situations, which cause great difficulty to math students and certainly is the reason why so many people say they hate math. In an attempt to justify these odd situations, the Traditional Math Model offers and accepts some poor explanations and theories, as the existence of complex numbers, equations with imaginary roots, equations with strange roots, rule of signs, and other questionable explanations.

The New Math Model provides the same "useful results", but is free from all these oddities and does not require poor explanations. Moreover, it is reasonable to expect that the logical reasoning and rationale behind the proposed model will favor students' performance when attending math courses, as well as people's appreciation for the science of math in their daily activities.

In order to demonstrate the above statements, I will summarize how each math model deals with those basic premises, to which they assign different meanings. It is not my intention to extend the effects of said premises to other math segments, a step outside and beyond the scope of this book.

Figure VIII.1 summarizes the innovative premises applicable to the New Math Model, and how different they are, when compared to their corresponding premises under the Traditional Math Model.

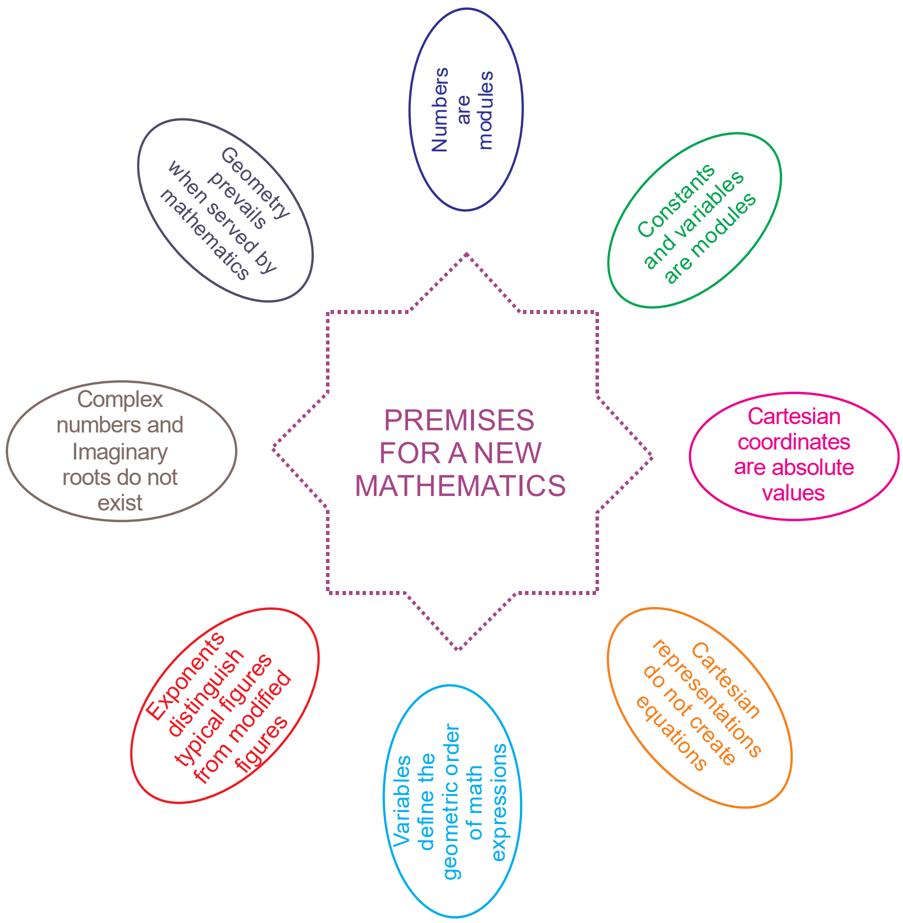
Numbers

The Traditional Math Model accepts the premise that we deal with positive and negative numbers, and provides different treatments to them. These treatments require the existence of imaginary numbers, a concept of questionable meaning and importance, and the assumption that exists a rule of signs that applies to multiplication, division and other operations with numbers and numbers behind letters in algebraic expressions.

The New Math Model adopts the concept that numbers in math expressions are neither positive nor negative, but neutral elements (absolute values or modules). As a result, complex and imaginary numbers do not exist, and we do not need the so-called rule of signs.

When dealing with math expressions, whatever the operation we perform, addition and subtraction are the sole commandments of the plus (+) and the minus (–) signs.

Figure VIII.1: Innovative math concepts



Constants and variables

The Traditional Math Model considers that letters, either constants or variables, in algebraic expressions representing numbers are also positive or negative elements.

The New Math Model treats constants and variables in math expressions in a similar way it deals with numbers. Constants and variables are absolute values or modules, what implies the interpretation of the results when dealing with forced equalities and accepted conventions.

Cartesian representations

Under the Traditional Math Model, the Cartesian System contemplates positive and negative coordinates, it being a consequence of accepting the premise that numbers are positive or negative elements. The Traditional Math Model also understands that the algebraic expression that represents a geometric figure placed in the Cartesian graph (as a parabola) creates an equation when the ordinate is equal to zero. It accepts that any "n" degree algebraic expression has "n" real or imaginary roots. It also accepts there are irrational equations with strange roots, as well as systems of equations.

Under the approach adopted by the New Math Model, the Cartesian coordinates are absolute values. In addition to that, it states that equations and Cartesian representations of geometric figures are two different math matters. As an example, when in the parabola Cartesian formula we make "$y = 0$", we do not create any equation. We simply determine points of the parabola curve located in the abscissas axis (if any). Cartesian expressions do not have real, imaginary or strange roots. The so-called system of equations[25] is nothing else but a ludic activity.

Geometric order and algebraic degree

The Traditional Math Model accepts extra dimensions (above the three dimensions accepted by geometry), as dictated by the algebraic degree of the math expressions. This includes polynomials of any degree, considered as an important math subject, with many practical applications.

The New Math Model states that the number of variables in an algebraic expression dictates the geometric order, which cannot go beyond the three dimensions in the real world. The exponents of the variables allow us to distinguish typical geometric figures (exponents equal to "1" and/or "2") from modified geometric figures (either integer or fractional exponents).

25 Except in case of scientific formulas.

The New Math Model understands classical polynomials as random algebraic expressions, representing flat curves in the Cartesian graph, which, with a few exceptions (as straight lines and parabolas), have no useful meaning or practical application.

Complex numbers and imaginary roots

The Traditional Math Model accepts the existence of complex numbers and imaginary roots of equations, since it understands that numbers are positive or negative elements, and that negative numbers require a different treatment.

The New Math Model does not recognize that concept of numbers, and consequently does not accept the existence of complex numbers or of imaginary roots of equations.

Geometry and mathematics

As the Traditional Math Model suggests, some people understands that geometry is a branch of mathematics, and that geometry must comply with math rules.

The New Math Model states that geometry is an independent science, which deals with a pre-existing subject of studies, the geometric figures. Geometry is a perfect creation of Mother Nature. Math is a tool, which serves other sciences, an imperfect creation of human being. With the help of algebra, calculus and other theories, methods and tools, math may deal with geometry under different points of view, but in case of a conflict between the rules of the two sciences, the geometry laws will always prevail.

Consistency of results

The Traditional Math Model will perform satisfactorily when dealing with practical applications, as the determination of the properties of geometric figures (lengths, areas and volumes), and the description of the behavior of the scientific phenomena (physics, astronomy, and others). It also performs well in relation to certain math areas (probability and combinatory analysis), which are not dependent upon certain algebraic concepts in use.

Within the algebra's realm, things may be quite different when using the Traditional Math Model, since we face odd operations, strange results, unproven conjectures, unsolved problems, and other unclear situations, not yet satisfactorily explained.

The New Math Model will perform satisfactorily in all cases, either practical applications or theoretical matters[26]. We will be free from odd arithmetic operations (as a multiplication of numbers with signs or the power or root of a negative number), and improper results (as a cube with a negative volume). If compared to the Traditional Math Model, the New Math Model will perform much more satisfactorily.

Simplicity

The Traditional Math Model coexists with questionable premises and unclear matters by simply accepting extravagant explanations, most of them previously referred in this book.

The New Math Model establishes consistent premises, which clarify the unclear matters and avoids the extravagant explanations by using logical reasoning.

Perhaps the most outstanding difference is the incomparable logic and simplicity of the rationale behind the New Math Model.

Advanced math

The innovative concepts applicable to the New Math Model will certainly cause some impact on subjects covered by advanced math. However, as I previously stated, this aspect is beyond the scope of this book.

26 Save the two math questions previously referred, the use of real numbers (non-terminating decimals) and the use of zero as a common number (undefined results)

CHAPTER IX

REASONS TO ADOPT THE NEW MATH MODEL

The strange results yielded by the Traditional Math Model, and the poor explanations if offers to accept said results, would be enough to justify the search for a better math model.

Preamble

At the beginning of this book, I questioned myself what would be the reasons to replace a millennial math model, which yields useful results, by another math model, which will yield the same useful results, but will require a drastic change in our way to see, interpret and use math.

In the pages that followed that question, I presented arguments, which in my view justify said approach, by suggesting some new math fundamentals, including a new concept of numbers, a Fundamental Axiom of Mathematics, and an Alternative Cartesian System. In fact, a New Math Model.

Logical reasoning as the rationale, and simplicity as the target.

Main reasons

As a summary of the reasons to accept said innovative math concepts:

(i) We will avoid those odd operations we see when using the Traditional Math Model, as the multiplication of numbers and terms with signs, the power of positive or negative numbers, and similar questionable arithmetic or algebraic operations.

(ii) Since we will not face odd operations, we will no longer need the so-called "rule of signs" or priority operating rules. We would handle arithmetic and algebraic operations (even involving terms with signs) as commanded by additions and subtractions, only.

(iii) The New Math Model makes a clear distinction between equations and the algebraic representation of geometric figures in the Cartesian graph, as well as (based upon the concept of proper and improper equalities) between absolute values and positive values in math equalities.

(iv) The field of algebra will be more restricted than we always thought. We will have a much better understanding about the use of the Cartesian System by algebra.

(v) In dealing with the Cartesian System, coordinates will be absolute values, and a single Cartesian quadrant will be enough to express any geometric figure in terms of algebra, unless we want to perform ludic exercises.

(vi) We will no longer face the inconsistencies found when dealing with the Traditional Math Model presently in use, as imaginary numbers, an equation with a strange root and similar oddities.

(vii) When dealing with practical matters, we do not have to find the square root of a positive or a negative number, and we may disregard the theory of complex numbers, and all further developments relating to it.

(viii) Some math conjectures and problems, which have been challenging mathematicians for centuries, may simply disappear as meaningless statements, resulting from inconsistent assumptions.

(ix) The proposed approach is much closer to the real world, because we will get rid of strange results and their poor explanations accepted by algebra under the Traditional Math Model, as a cube with negative volume.

(x) This logical approach will lead us to an easier discipline, improving student's performance when attending math courses.

(xi) Additionally, that new approach will certainly minimize people's dissatisfaction with math matters.

The acceptance of the New Math Model I propose in this book will imply a full conceptual revision of the science of math, to establish a new way to see and handle its fundamentals, tools and methods, as well as to how to interpret and explain math results. The required revision will have to start from the very beginning of math foundation, the "concept of numbers". However, it seems the task is worth the effort.

After introducing the New Math Model, in the following pages I will discuss three famous math subjects: Pythagoras' Theorem, Fermat's Conjecture (Fermat's Last Theorem, after Andrew Wiles), and the Millennium Problems.

CHAPTER X

PYTHAGORAS' THEOREM

Pythagoras' Theorem is the most famous, and possibly the oldest, math theorem.

According to the present understanding, this Theorem is a property of the right triangle. I say it is a property of the circumference and has a much broader meaning than we always thought.

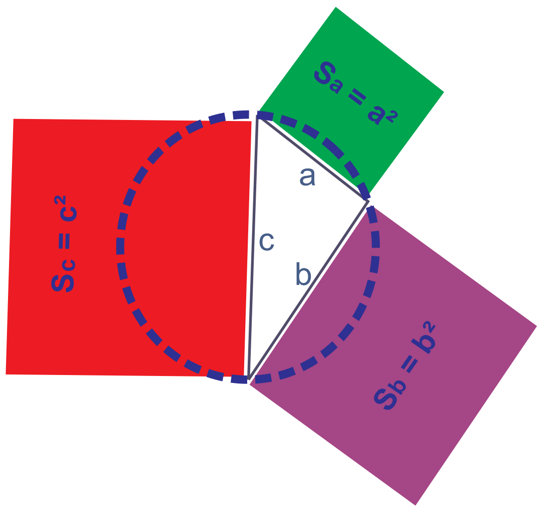
Traditional concept

If Pythagoras' Theorem is not the oldest one, for sure it is the most famous theorem of mathematics. It has such name to honor the Greek mathematician Pythagoras, who lived around five centuries before Christ, although many people argue that the theorem is much older, perhaps known since the ancient Mesopotamian civilizations.

Whatever, this theorem, illustrated in Figure X.1, expresses a property of the right triangles in its millennial and true statement that "In any right triangle, the sum of the squares of the legs is equal to the square of its hypotenuse", a statement known as the Pythagorean relationship.

The literature informs there are hundreds demonstrations for that theorem, including the ones proposed by the Greeks Euclid and Archimedes, besides those of Pythagoras, Bhaskara, da Vinci, and the United States president, James Abram Garfield. These demonstrations rely on different backgrounds, and I do not know it any of them is a purely algebraic one[27]. I will show that the quadratic relationship that applies to the right triangle, as stated by Pythagoras' Theorem, has a broader meaning and is a property of the circumference chords.

Figure X.1: Pythagoras' Theorem



In a similar way I questioned some math fundamentals, I will question the prevailing

27 It seems it is not possible to demonstrate this theorem under a solely algebraic conception.

understanding of certain math matters, as Pythagoras' Theorem, and Fermat's Conjecture (Fermat's Last Theorem after Andrew Wiles).

I will show that the quadratic relationship between the right triangle and the circumference allowed me to suggest and prove a new theorem, which I named "Theorem of Infinite Right Triangles".

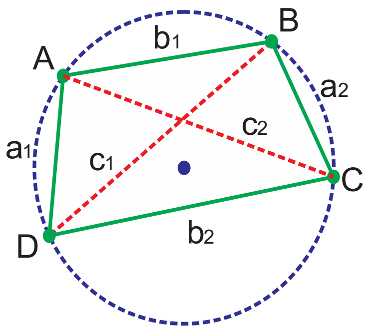
Ptolemy's Theorem

To introduce the subject, it is convenient to recall a theorem attributed to Claudio Ptolemy about quads inscribed in a circumference, which says: "The product of the diagonals of a quad "ABCD", inscribed in a circumference, is equal to the sum of the products of the opposite sides of the quad (chords)", as shown in Figure X.2.

In algebraic terms:

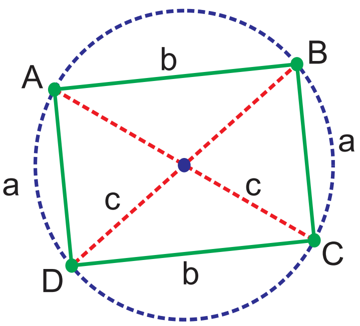
$$a_1.a_2 + b_1b_2 = c_1.c_2$$

Figure X.2: Ptolemy's Theorem



By observing Figure X.3, we see that when the two chords are parallels and equidistant from the center of the circumference, the quad becomes a rectangle, and its diagonals are also diameters of the circumference, what means that "$c_1 = c_2 = c =$ $= 2r$". Such rectangle contains the right triangles "ABD" e "BCD", and since "$a_1 = a_2 =$ a" e "$b^1 = b^2 = b$", then "$a^2 + b^2 = c^2 = (2r)^2$". Pythagoras' Theorem becomes a special case of Ptolemy's Theorem.

Figure X.3: Pythagoras' Theorem as a special case of Ptolemy's Theorem



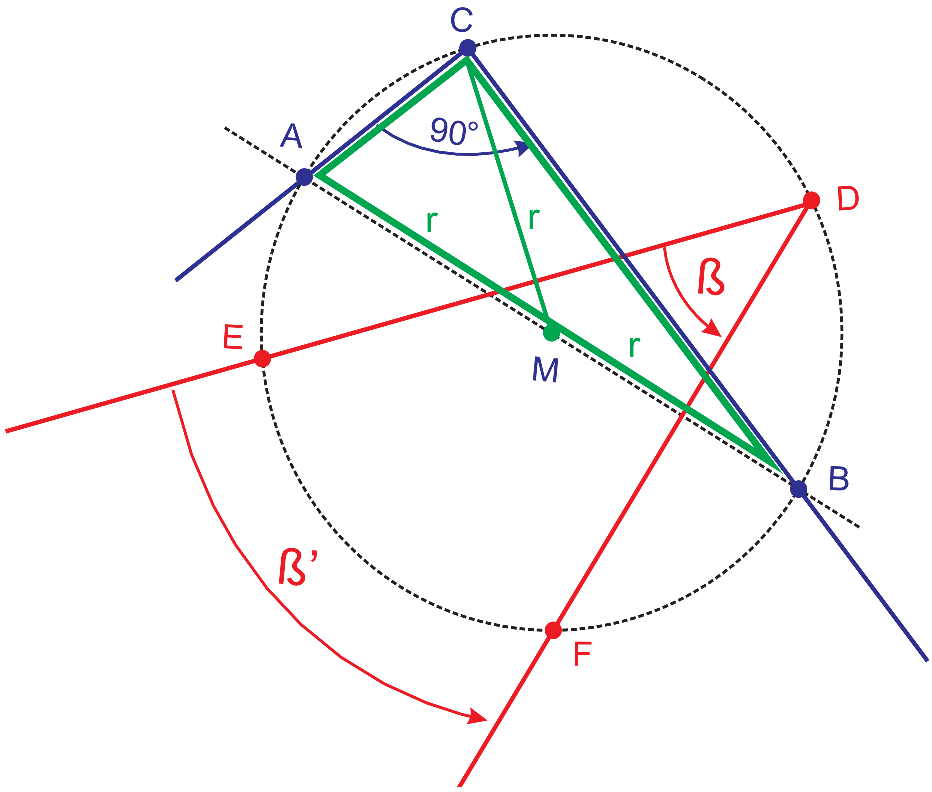
Pythagoras's Theorem is a particular case of Ptolemy's Theorem.

In view of that, my understanding is that Ptolemy's Theorem is not a property of a quadrilateral. It is a property of the circumference or, more specifically, a property of the chords of the circumference.

The chords of a circumference and its quadratic relationship

The angle "β" formed by secants "DE" and "DF" from a common point "D" placed in the circumference, shown in Figure X.4, is equal to half of the arch "β'" defined by such secants in the circumference, whatever the value of arch "β'".

Figure X.4: Angle formed by secants and the inscribed right triangle

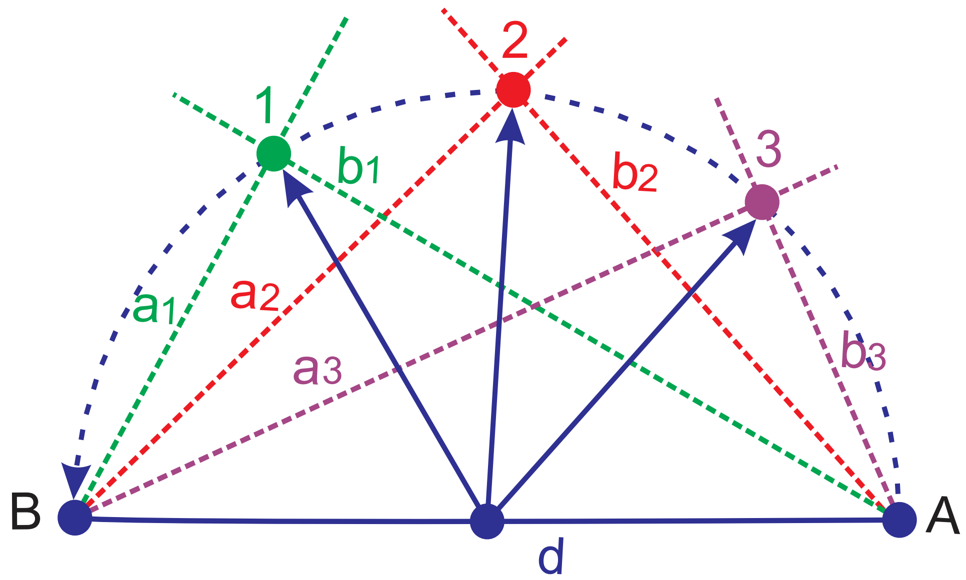


It is then clear that two secants, which connect a point "C" in the circumference to the sides of any diameter, for instance "AB", define an arch of "180°" in the circumference, what means that the angle "ACB" will always be a right angle (90°).

As a result, the triangle formed by chords "CA" e "CB" with the diameter "AB" is a right triangle for any location of point "C" (with the sole exception when point "C" coincides with the ends of the diameter "AB"), and the hypotenuse is always the diameter of the circumference.

Since all the infinite triangles formed as from any point "C" in Figure X.4, as well as from any points 1, 2, 3 … in Figure X.5 are right triangles and have the same hypotenuse "AB", Pythagoras' Theorem requires that in all those infinite triangles their legs may vary provided the sum of their squares remains constant.

Figure X.5: Right triangles



Theorem of the Infinite Right Triangles

We then may enunciate the previously referred quadratic relationship, as a property of the circumference:

> **The sum of the squares of two chords that connect any point in a circumference to the ends of any diameter is constant and equal to the square of said diameter or four times the square of the circumference radius.**
>
> $$a_1^2 + b_1^2 = a_2^2 + b_2^2 = a_3^2 + b_3^2 = d^2$$

Moreover, expressed as a function of the right triangle, that quadratic relationship allows us to state the Theorem of Infinite Right Triangles:

> **Any straight-line segment is the hypotenuse of infinite right triangles, in which the legs may take any value, provided the sum of their squares remains constant.**

As a result, we may state the Theorem of Infinite Right Triangles in a more general algebraic form:

> $$(a^2 + b^2) = \text{constant}$$

Other authors have already referred to the statement that the sum of the square of the legs is a constant, but to my knowledge, always as a property of the right triangle, and not as a property of the circumference. In addition to that, without recognizing the real thing:

> **The Theorem of Infinite Right Triangles is a generalization of Pythagoras'**

Theorem.

Clearly, a same straight-line segment is the hypotenuse of an infinite number of right triangles, and the square of that segment is equal to the sum of the squares of the legs of all those infinite right triangles. The square of that segment also is equal to four times the square of the radius of the circumference in which the right triangles are inscribed.

Corollaries of the Theorem of Infinite Right Triangles:

First Corollary:

The geometric place described by the vertexes of the right angles of the infinite right triangles formed as from a same hypotenuse is a circumference.

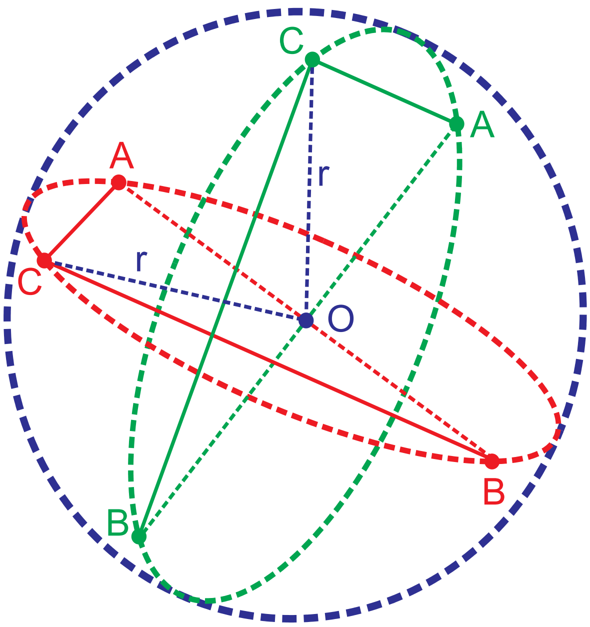
We may express this first corollary by a broader statement, by saying that each circumference contains infinite families of geometric figures, each one with infinite members, as triangles, squares, rectangles and other uncountable geometric figures, all of them tied to the radius of the circumference.

Second Corollary:

Any point in the surface of a sphere forms an infinite number of right triangles when connected to the ends of any of the infinite sphere diameters.

This second corollary, illustrated in Figure X.6, results from the fact that a circumference contains infinite right triangles and the sphere contains infinite circumferences, with a unique diameter, the sphere diameter.

Figure X.6: Infinite right triangles in a sphere



The reasons why I say that the quadratic relationship, although perfectly applicable to any right triangle, is a property of the circumference, not a property of the right triangle are as follows:

(i) It is possible to generate a circumference by rotating squares, rectangles, triangles and many other geometric figures that, similarly to the case of the right triangle, all of them inscribed in the circumference (see Figure X.7).

(ii) As a result, we may say that the circumference exists without the right triangle, but the reciprocal statement is not true, since no right triangle may exist outside a circumference (see Figure X.9).

(iii) Whenever a point in a circumference, which is the vertex of a right angle of any of the infinite right triangles generated as from any diameter taken as the hypotenuse, coincides with one of the ends of said diameter, the quadratic relationship prevails, although no right triangle exists.

(iv) Seen isolated, any right triangle shows three constant sides in such a manner that the condition that the legs may vary for the same hypotenuse is hidden, is implicit, what does not happen when the right triangle is seen inscribed in the circumference.

The conclusion:

> **The Theorem of Infinite Right Triangles, a property of the circumference, is a generalization of Pythagoras' Theorem, the latter always seen as a property of an individual right triangle.**

Applications of the Theorem of Infinite Right Triangles

The circumference is a flat geometric figure and follows a 2nd-order geometrical law, represented by a 2nd-degree algebraic expression. That is the reason why Ptolemy's Theorem, Theorem of the Chords, Pythagoras' Theorem, and the Theorem of Infinite Right Triangles are 2nd-order algebraic expressions, represented by 2nd-degree algebraic expressions.

The extended view of Pythagoras' Theorem to encompass other areas of similar shapes built from the sides of right triangles, as polygons, circles and others, remains valid for the Theorem of Infinite Right Triangles.

In a previous work, I used the Theorem of Infinite Right Triangles to demonstrate Fermat's Conjecture (Fermat's Last Theorem, after Andrew Wiles), putting together geometry and algebra, since the two subjects follow 2nd-degree math expressions.

We may understand the square in Figure X.7(b) and the rectangle in Figure X.7(c) as formed by right triangles as in Figure X.7(a), what shows a connection among these polygons and of each of them with the circumference.

It is worthwhile to point out that Pythagoras' Theorem and the Theorem of Infinite Right Triangles work for integer and non-integer numbers as well (the latter as approximate relationship). In general, we may call these numbers as quadratics, and if they are integers, as Pythagorean numbers. Figure X.8 illustrates these two situations.

Figure X.7: Application of the Theorem of Infinite Right Triangles

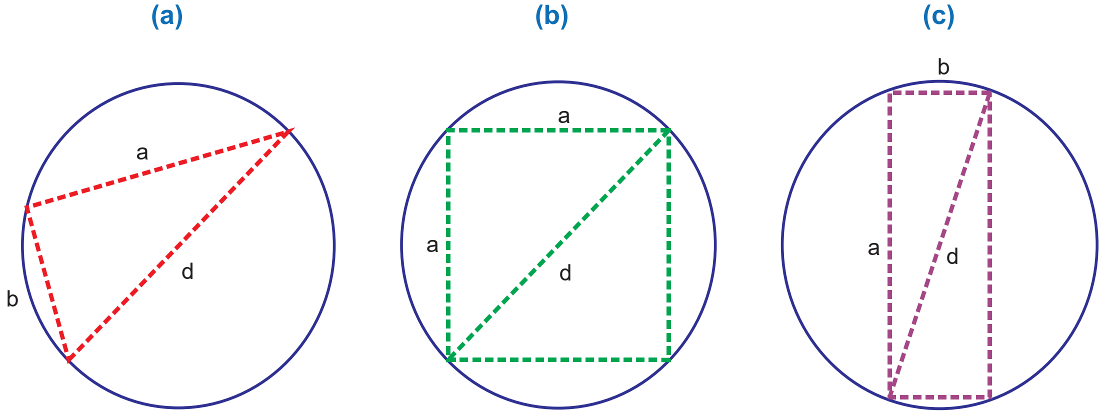
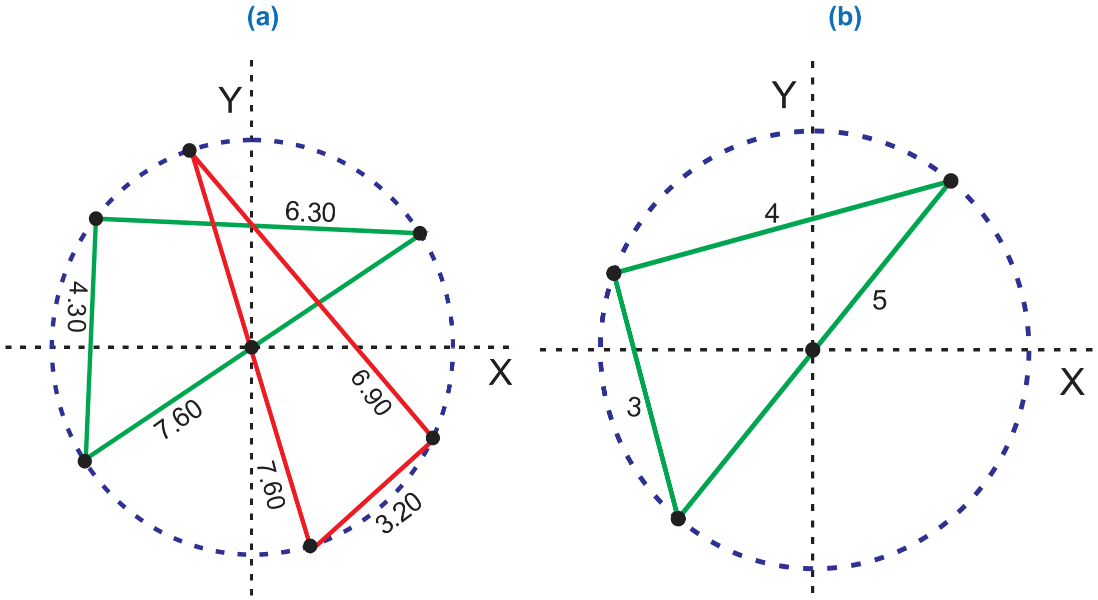
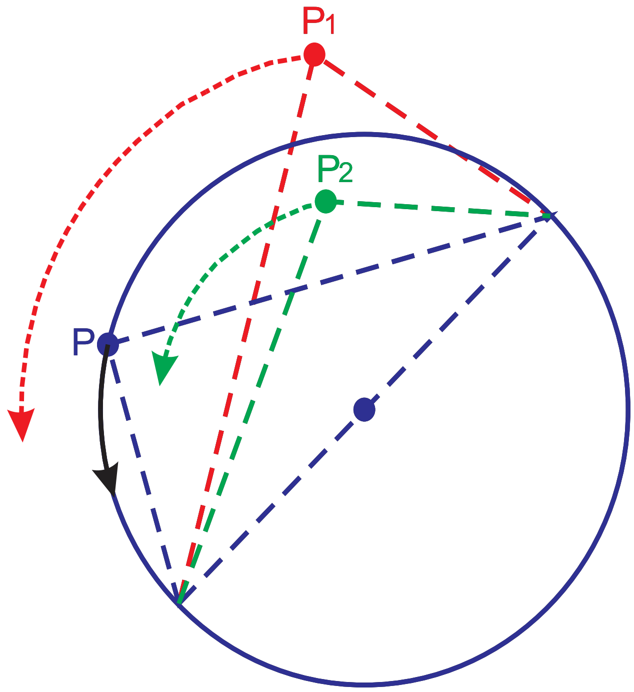


Figure X.8: Pythagorean and non-Pythagorean numbers



There are some interesting relationships between triangles and the circumference, as shown in Figure X.9. A point "P" located in the circumference meets the quadratic relationship and forms a right triangle with any diameter. This means that such point "P" may rotate in relation to the center of the circumference and generate infinite right triangles, all of them with the same hypotenuse. However, points outside the perimeter of the circumference (external as "P_1" or internal as "P_2") do not relate to that circumference by means of a quadratic relationship. Their connections to the ends of any diameter of the circumference are not chords. They are segments, which will form acute (from "P_1") or obtuse (from "P_2") triangles.

Figure X.9: Violation of the Pythagorean relationship



The reciprocal of these two theorems is a valid statement: “Three straight-line segments represented by three numbers different from zero, which meet the quadratic relationship necessarily form a right triangle”.

CHAPTER XI

FERMAT'S CONJECTURE (FERMAT'S LAST THEOREM)

The proof presented by Andrew Wiles, and recognized by the math community, refers to the elliptical curves. I say that Fermat's Conjecture has a connection with the hyperellipse.

Algebra alone cannot demonstrate Fermat's Conjecture, because this Conjecture refers to a geometric impossibility.

History

Almost 400 years ago, when reading the book "Diophantus Arithmetic", Pierre de Fermat proposed a conjecture, which states:

> **With whole numbers, it is impossible to express a third power as the sum of two third powers or a fourth power as the sum of two fourth powers or, in general, any number to a power greater than the second power as the sum of two powers with the same exponent.**

Fermat also added:

> **I found a truly marvelous demonstration for that statement, but the margin of this book is too narrow to write it.**

The conjecture initially known as Fermat's Conjecture became Fermat's Last Theorem after the demonstration made by the British mathematician Andrew Wiles. If Fermat in fact had a proof for his Conjecture, nobody knows, since he did not disclose it. Fermat's Conjecture is not famous because it has practical applications, but due to the fact that it challenged mathematicians for almost four centuries.

Pierre de Fermat, a French judge, gave a significant contribution to mathematics, the reason he is known as the "The prince of the amateurs". Fermat was used to making annotations to the margins of the books he uses to read. After his death, his son Clémant Samuel published some of those annotations, including the one written in the book "Diophantus Arithmetic", in which he stated he had a truly marvelous demonstration that there are not integer numbers that meet the relationship "$x^n + y^n = z^n$", for "$n > 2$", but the book's margin was too narrow for him to write it.

Evidently, Fermat's note suggests that his prove, if it in fact exists, was rather concise and ingenuous. Nobody knows if Fermat had the demonstration, but to find it became a challenge for mathematicians and non-mathematicians as well.

Uncountable demonstrations have been proposed, and as a result nowadays nobody wants to hear about Fermat' Conjecture. Since being proposed, the best mathematicians in the world unsuccessfully tried to find Fermat's proof. If such demonstration does exist, it must be different from the demonstration proposed by Andrew Wiles, who work for many years and used numerous equations and theories that Fermat did not know at his time. Although rather useless for practical purposes, no other math problem possibly received so much attention as Fermat's Conjecture did. The challenge last so long that it became an obsession for mathematicians and non-mathematicians, as Wiles, who devoted years searching for the Conjecture proof.

Some people have the belief that Fermat was mistaken and had no demonstration for his own Conjecture, at least one we could call "a truly marvelous demonstration" as referred to by Fermat. This doubt is supported by the fact that among his post-mortem published notes there is a trial to prove the Conjecture for "$n = 4$".

After Fermat's death, the math community accepted that some mathematicians succeeded in proving the Conjecture for "n" equals to "3", "4" and other exponents. The use of modern computers made it possible to extend the search, and it never found three numbers to contradict Fermat's Conjecture.

The famous Indian mathematician Srinivasa Ramanujan suggested numbers pretty close, but not enough to contradict the Conjecture. He identified a way to generate numbers of near misses of the form:

$$x^3 + y^3 = z^3 \pm 1$$

$$10^3 + 9^3 = 12^3 + 1$$

$$1000 + 729 = 1728 + 1$$

Following the speculative character of this book, of questioning traditional math concepts and results, I will offer a new way to look at Fermat's Conjecture.

Demonstrations

In a previous book[28], I offered a proof of Fermat's Conjecture with a demonstration that started with the new concept for Pythagoras' Theorem (the Theorem of Infinite Right Triangles, already explained), and a reasoning about the triangles we could form with three straight-line segments. In a subsequent paper, I offered another demonstration for Fermat's Conjecture, which I reproduce here.

In fact, I assumed from the starting point that it is not possible to demonstrate Fermat's Conjecture solely by algebraic means. This problem requires the

28 AMUI, Sandoval, "A Circunferência, Pitágoras e Fermat, Editora CataLivros, Rio de Janeiro/RJ, 2019.

combination of algebra and geometry concepts. In both demonstrations I offered, I used the idea of the American-Canadian mathematician Robert P. Langrands, who suggests the association of different math areas to solve math problems.

When talking about Fermat's Conjecture, mathematically related to the algebraic expression "$x^n + y^n = z^n$", we must keep in mind we are not able to express any figure with an infinite number of decimals. Mathematically speaking, only the Pythagorean right triangles are true right triangles; all others are approximations.

By rewriting the above-referred math expression "$x^n + y^n = z^n$", as "$(x^{n/2})^2 + (y^{n/2})^2 = (z^{n/2})^2$", we may say that, in order to be a true equality (exact figures), that math expression must represent a circumference. It also implies that "$x^{n/2}$", "$y^{n/2}$" and "$z^{n/2}$" should be whole numbers and "$n = 2$", in a manner that "$x^{n/2}$" and "$y^{n/2}$" would be the legs, and "$z^{n/2}$" the hypotenuse of a Pythagorean right triangle.

Wiles' demonstration deals with the elliptical curves, while in my view Fermat's Conjecture has a relationship with the hyperelipse (a particular case of the superellipse when "n" is greater than "2"). In view of that, let us analyze that specific geometric curve.

The Cartesian expression of a superellipse is:

$$(x/a)^n + (y/b)^n = 1$$

If "$a^n = b^n = z^n$", then the expression becomes "$x^n + y^n = z^n$", and the conclusion is that Fermat's math expression is a particular case of a superellipse, as seen in Figure XI.1 below:

Figure XI.1: Superellipse

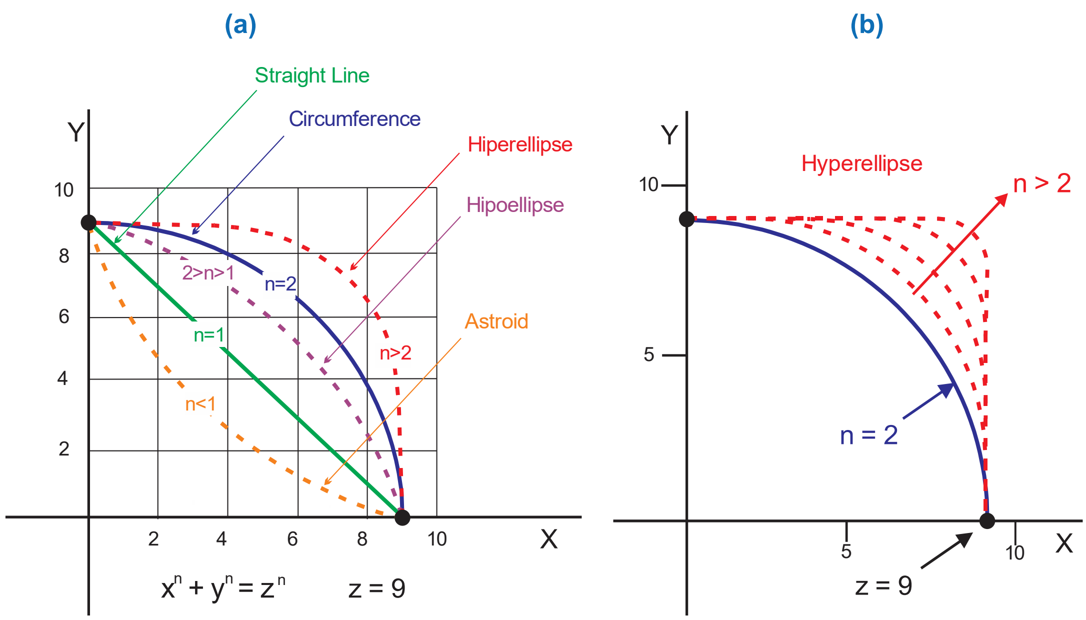


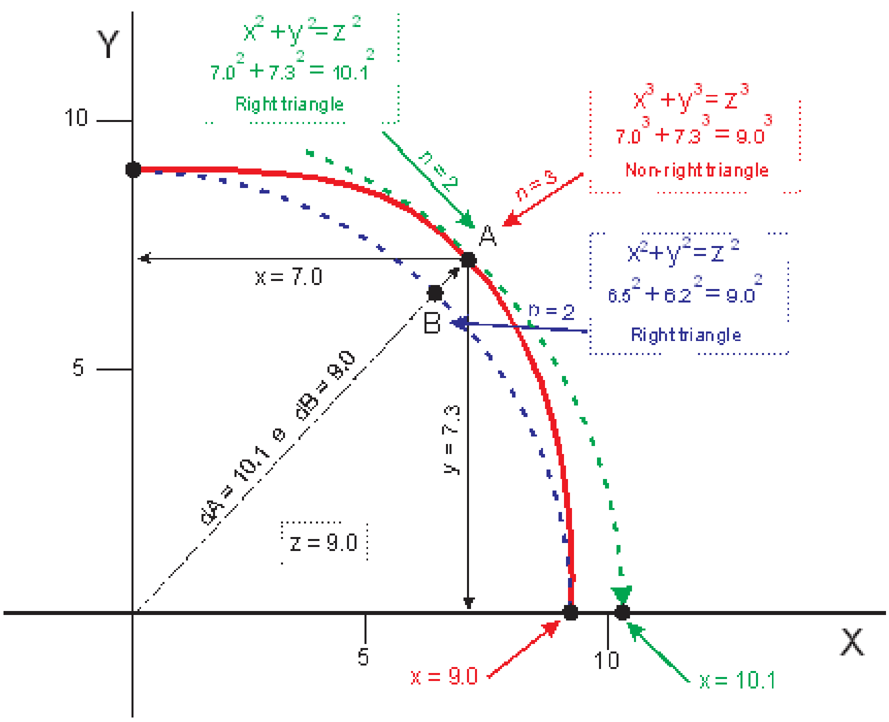
Figure XI.1(a) illustrates the superellipse, a geometric figure also called Lamé's Curve or rounded square. In the form "$x^n + y^n = z^n$", we see that it has as particular cases the straight line ($n = 1$), the circumference ($n = 2$), the hypoellipse ($1 < n < 2$), and the hyperellipse ($n > 2$). When "$n = 2/3$" the curve is named astroid.

Figure XI.1(b) indicates that when "n" increases, keeping "z" constant, the curve tends to a square with rounded corners, it being the reason the superellipse is also known as "rounded square".

The math expression "$x^n + y^n = z^n$", which represents Fermat's Conjecture, is a particular case of a superellipse when "n" is greater than "2", as in Figure XI.2.

Said math expression corresponds to a circumference when "$n = 2$" and "z" is the radius, "$r = 9$", as well as to a hyperellipse when "$n = 3$" and "$z = 9$". We see that in order to contradict Fermat's Conjecture, we would need a Pythagorean right triangle with two different hypotenuses or a circumference with two radius represented by two different whole numbers. Two geometrical impossibilities.

Figure XI.2: Circumference and hyperellipse



My analysis about Fermat's Conjecture in connection with the superellipse and the relationship between the circumference and the right triangle allows me to state that:

> **The right triangle that could contradict Fermat's Conjecture does not exist. In order to contradict Fermat's Conjecture we need a Pythagorean right triangle with two different hypotenuses or a circumference with two different radius expressed by integer figures. Obviously, two geometric impossibilities.**

CHAPTER XII

UNPROVEN CONJECTURES AND UNSOLVED PROBLEMS

Unproven conjectures and unsolved problems we still face in math may be improper statements, resulting from the acceptance of inconsistent premises.

The millennium problems

As from the year 2000, the Clay Mathematics Institute offers a US$ 1 million prize to anybody who correctly[29] solves any of the seven problems known as the "Millennium Problems": Birch and Swinnerton-Dyer Conjecture, Hodge Conjecture, Navier-Stokes Equations, P versus NP Problem, Riemann Hypothesis, Yang-Mills Problem, and Poincaré Conjecture.

In 2003, the math community (including the Clay Institute) recognized that a Russian mathematician (Grigori Perelman) solved the Poincaré Conjecture, but Perelman declined the Clay prize. The other six problems remain unsolved.

There are much more than six unproven conjectures and unsolved problems, a long list, which started with 23 math problems put together by the German mathematician David Hilbert in 1900. Since Hilbert, said list increased due to new conjectures and problems. From time to time, the math community accepts a proof as a correct solution for one of these challenges. Nevertheless, the list remains a long one.

In line with the questioning approach adopted in this book, I already mentioned my assumption that some of these unproven conjectures and unsolved problems may only be inappropriate statements, based on inconsistent premises, including some of the six Millennium Problems. If that assumption is true, it is not possible to prove conjectures or solve problems, which do not exist.

Examples

To illustrate my assumption, let us consider two examples of the Millennium Problems:

The math structure in force accepts the existence of positive and negative numbers, and that, in performing operations, the operating rules applicable to negative numbers differ from the operating rules applicable to positive numbers. This

29 The first difficulty is to understand the meaning of each one of these problems. The second difficulty, if you are able to overcome the first one, is to meet all Clay Institute requirements to consider that a proposed solution is an acceptable trial.

fundamental concept leads to a questionable theory about “complex numbers”, which accepts the existence of imaginary numbers.

The Riemann Hypothesis is a question that deals with complex numbers. If, as I say, this theory of complex numbers is a mistaken approach, the challenge behind the Riemann Hypothesis may not exist either.

Math concepts in use accept many math expressions as equations, including the Diophantine expressions. The Conjecture proposed by Birch and Swinnerton-Dyer is the tenth problem in Hilbert’s list. Hilbert wanted an algorithm to find integer roots for these so-called Diophantine equations. The Birch and Swinnerton-Dyer Conjecture deals with that subject.

It is possible to find integer values that satisfy the math expressions accepted as Diophantine equations, particularly certain expressions, when “$n = 1$” (as points of a straight line) and “$n = 2$” (as the sides of Pythagorean right triangles).

However, in accordance with the restricted meaning attributed to equations in this book, “Diophantine equations” are not equations, and the task to “solve” them is part of ludic algebra. They are interesting exercises with a few (if any) practical applications.

As I already said, I am not a mathematician, and I do not have the required knowledge or the intention to discuss any one of these Millennium Problems. I am not even prepared to understand what these unproven conjectures and unsolved problems actually mean. I simply say that,

> **If we adopt different math fundamentals, as the meaning of numbers and related subjects, we have to look at math matters (including unproven conjectures and unsolved problems) in a different way.**

FINAL WORDS

The innovative assumptions and concepts I offered in this book, as a New Math Model, are undoubtedly polemic and, if adopted, will lead to a different math structure.

My assumptions may be wrong or they may be right; this is an open question.

Math concepts under the Traditional Math Model yield unquestionable oddities; that is a certainty.

Most students do not achieve in math the same academic results they usually do in other disciplines. Faced at an early school stage, this learning difficulty propagates throughout their entire lives. Math then becomes the least appreciated discipline in almost every school, and people often say they hate math.

Different reasons have been listed to explain such undesirable situation, but except for some few and timid criticisms, nobody clearly states that the reasons may be within math itself. People interested in math may find books and videos on the internet always teaching the same things in the same way, but I did not see anyone with a different approach, offering a new thesis or at least trying to explain the many strange results yielded by the current math structure.

As any other area of studies of human knowledge, math relies on postulates, premises, axioms and theories accepted as valid statements, what may not be the case. Undoubtedly, this field of work coexists with inconsistencies, poorly explained results, undefined and indeterminate forms, unproven conjectures and unsolved problems, what does not make sense in a field of work considered as an “exact science”.

I limited my analysis to the fundamentals of elementary math, since my suggestions stay in the field of math basic premises. Nevertheless, I offered a few practical examples in Appendix “A”, to demonstrate the feasibility of my propositions. They worked perfectly.

I offered here some truly polemic propositions, which once accepted will drastically change our way to see and handle math subjects. These propositions may be wrong, of course. However, they may be correct. Some math results we obtain when we apply the concepts in force are clearly mistaken, and it is a good reason to consider that at least math needs and deserves a critical revision.

It seems we need a different mathematics, better aligned with the real world and tailored to suit the day-to-day needs of any common person. A new approach that would yield the same useful results, without the inconsistencies, strange responses, and poorly explained questions found in connection with the Traditional Math Model. A new math foundation that favors the enjoyment of math and the improvement of students’ performance.

I dare to state that it is possible to achieve this target by simply assigning different interpretations to some of the existing math fundamentals as I discussed in this book.

This new approach about math premises and concepts may have significant impact and effects on the way we interpret and deal with math expressions. Even if they are accepted and implemented, math will still need further revisions and improvements in addition to the ones here proposed.

As a language to serve other areas of human knowledge, and contrarily to geometry, physics, astronomy and other sciences, math does not have a pre-existing and proper subject of studies, other than its own premises, rules and methods. Without violating its valid principles, math must comply with the laws ruling the subject of the area it serves. Geometry is not a branch of math, but an independent area of studies. Math may deal with geometry under different approaches, but in all cases, math must comply with the laws of geometry.

The math representation of nature continuity is perhaps the most important problem that challenges mathematicians, since non-terminating decimals only yield approximate values. True values only occur when input data and results are whole numbers; Math needs a better way to represent the continuity that exists in nature, other than the present concept of real numbers.

Unclear situations involving the number zero is a common difficulty faced by math students, particularly when dividing a real number by zero or dealing with the limit of indeterminate forms. However, limit indeterminate forms are a peculiarity of certain invalid algebraic expressions and the misuse of the Cartesian System. It would be convenient to review the properties and uses of the number zero, a matter that deserves further thoughts. The use of the number zero as a common figure is awaiting for clarifications and improvements.

Some proposed theories, as well as unproven conjectures and unsolved problems may simply be improper statements resulting from imperfect math premises and assumptions. Building cracks resulting from structural defects.

In brief, I suggest the revision of certain math fundamentals, which support the math structure in use (here named Traditional Math Model), and their replacement by more consistent premises, which lead us to a different math structure (here referred to as New Math Model).

This measure will require the revision of almost all math segments, beginning with the most fundamental math premise - the concept of numbers, with the purpose to identify and perform the needed adjustments from both standpoints, the theoretical understanding and the practical application of the new concepts to each one of these segments, a task outside the scope of this book.

Under a new interpretation of currently accepted premises, concepts and theories, algebra may lose some of its present charm, but it will be for the benefit of the whole of math. We will be free from the inconvenience of dealing with negative and complex numbers, rule of signs, strange roots of equations, indeterminate forms, and other unclear matters. We will deal with a much simpler and logic science.

I also suggest the updating of math programs and teaching approach so far adopted by schools to improve students' achievements regarding math courses. Under an approach simpler and more in line with the real world, the students will have a better understanding of math uses and limitations, and will improve their academic performance when attending math classes. It is reasonable to believe that these improvements may also favor the appreciation and avoid unpleasant feelings most people nurture towards math.

> **Math will no longer be the "intimidating monster" students will have to fight and hate.**

APPENDIX “A”

EXERCISES UNDER THE TWO MATH MODELS

The fact that the Traditional Math Model gives us useful results does not mean it is perfect and cannot receive improvements.

Introduction

It is not my intention to extend the effects of the innovative concepts of the New Math Model to the many math segments, except in respect of a few elementary math exercises, to illustrate that the practical application of these innovative concepts is feasible and yields results consistent with the real world.

In this Appendix, I will accomplish that by means of a comparative analysis of the results we obtain when solving these elementary math exercises under the premises of the Traditional Math Model and the New Math Model.

As a relevant remark, I must say that the useful results would be the same, but the rationale behind the operations, and the interpretations of the results would be different.

Proper and improper equalities

I introduced in Chapter V the concept of proper (true) and improper (false) equalities, which, in my view, the Traditional Math Model does not contemplate. A few arithmetical and algebraic examples may clarify the idea.

Arithmetic equalities

Find the value of “k” in the following arithmetic expression:

$$k = 5 - 3 - 7 + 2$$

Under the concepts of the Traditional Math Model:

$$+ k = + 5 - 3 - 7 + 2$$

$$+ k = - 3$$

In fact, we are saying that “$+ |k| = - |3|$”. This answer does not make sense, since we are accepting that a positive value is equal to a negative value. An inconsistent equality, resulting from an incorrect assumption that a positive value ($+ k$) is equivalent to an absolute value ($|k|$).

Under the premises of the New Math Model, we will obtain the same answer, but unless this result has any specific meaning and requires interpretation (as the answer to a pre-formulated question), we are aware that we deal with an improper equality, due to the acceptance of an incorrect assumption, since $+ |k| \neq - |3|$.

We obtained the same numerical result using the Traditional Math Model and the New Math Model. However, we interpret the results in different ways.

As another example, we may consider a power "a^n", in which "a" is a constant and "n" is an even or an odd exponent.

Under the Traditional Math Model, "a" must be a positive or a negative number, and we must use the rule of signs in force, which depends on the nature of the exponent (an even or an odd number), as follows:

$$+2^2 = +4; \qquad -2^2 = +4; \qquad +2^3 = +8; \qquad \text{and} \qquad -2^3 = -8$$

Under the New Math Model, numbers are neutral elements. Additionally, the plus (+) and minus (–) signs only command additions and subtractions. That is the reason to use a different notation. On purpose, under the New Math Model, I will separate the plus (+) and minus (–) signs from numbers and terms, exactly to show that they only mean addition and subtraction operations of these numbers and terms, what I did not do above, when applying the Traditional Math Model.

Then, in accordance with the New Math Model, the conceptual treatment of the same numerical examples above will be different:

$$+\ 2^2 = +\ 4; \qquad -\ 2^2 = -\ 4; \qquad +\ 2^3 = +\ 8; \qquad \text{and} \qquad -\ 2^3 = -\ 8$$

We see that, under the Traditional Math Model, the resulting power of a number commanded by a negative (–) sign depends whether the exponent is an even or an odd number ($-2^2 = +4$, while $-2^3 = -8$), what does not occur under the New Math Model ($-\ 2^2 = -\ 4$, and $-\ 2^3 = -\ 8$).

Under the New Math Model premises, "$-\ 2^2 = -\ 4$" means "$-\ |2|^2 = -\ |4|$", and "$-\ 2^3 = -\ 8$" means "$-\ |2|^3 = -\ |8|$". The plus (+) and the minus (–) signs in front of these powers only appear if they are terms of a math expression, as follows:

$$+\ y = +\ a^n + b \quad \text{or} \qquad +\ y = -\ a^n + b$$

We do not face the problem to raise positive or negative numbers to any power, whatever the power exponent. The plus (+) and minus (–) signs in front of a power, as "a^n", only mean the whole term considered (a^n), is an absolute value, which must be added to the class commanded by the same sign we see in front of it, either the plus sign ($+\ |a^n|$) or the minus sign ($-\ |a^n|$). Clearly, "a" and "n" are absolute values ($|a|$ and $|n|$).

Similarly, in case we will have to deal with the square root of a positive or a negative number (as in connection with ludic exercises), we must interpret it in the same way, as the square root of an absolute number, which will follows the plus (+) or the minus (–) commandments (addition or subtraction), as follows:

$+\sqrt{(-4)}$	**means**	**$+\sqrt{(-\lvert 4\rvert)} = -\lvert 2\rvert$**
$-\sqrt{(-4)}$	**means**	**$-\sqrt{(-\lvert 4\rvert)} = +\lvert 2\rvert$**
$+\sqrt[3]{(-27)}$	**means**	**$+\sqrt[3]{-}(\lvert 27\rvert) = -\lvert 3\rvert$**
$-\sqrt[3]{(-27)}$	**means**	**$-\sqrt[3]{-}(\lvert 27\rvert) = +\lvert 3\rvert$**

It is mandatory to clarify that, according to the New Math Model, and as I stated in Chapter V, these kind of operations with numbers and letters commanded by the plus (+) and the minus (–) signs will only occur when said numbers and letters are terms of math expressions (never as isolated terms).

Algebraic questions

Find the value of "k" in the following algebraic expression when "x = 3":

$$k = 5 - x - 9 + 2x$$

As seen under the Traditional Math Model, "x" may be a positive or a negative value, but, as per the prevailing convention, we know there exists an omitted plus (+) sign before "k" (+ k), as well as before "3" (x = + 3); then:

$$+k = +5 - 3 - 9 + 2(3)$$

$$+k = -1$$

The Traditional Math Model accepts that answer, "+ k = – 1", meaning that "+ k = |k|", a clearly mistaken equality, because that math model considers positive values as equivalent to absolute values.

Under the New Math Model, we know that "k" is an absolute value, which must be considered as belonging to the positive class of terms (but placed in the left side of the equality expression), and understand that "x" is also an absolute value, "x = |3|", (but placed in the right side of the equality expression), what lead us to:

$$+\lvert k\rvert = +\lvert 5\rvert - \lvert 3\rvert - \lvert 9\rvert + (\lvert 2\rvert)(\lvert 3\rvert)$$

$$+\lvert k\rvert = -\lvert 1\rvert$$

Under New Math Model, "+ k ≠ |k|", and we are facing an improper equality, an unacceptable algebraic expression, unless we have a meaning for that answer, a case in which we have improperly formulated the question we are dealing with.

Although possible, it does not make sense if we want “x” as a positive value (x = +|2|), or as a negative value (x = – |2|) in the above example.

The plus (+) sign in front of the term with “x” (“+ 2x” in the above example) already informs this whole term (2x) belongs to the class commanded by the plus (+) sign. *Mutatis mutandis*, the same reasoning applies to the minus (–) sign in front of any term of an equality. Logical reasoning simplicity.

As another example, suppose a certain bank client tells the bank teller that his account balance “z” is a credit (a positive value by convention), and asks the teller to check his account balance.

Based on the client’s information, the teller reasons that the client balance is “+ z = + x – y”, where “x” is the client’s total credit (a positive amount by convention) and “y” is the client’s total debt (a negative amount by convention). When verifying the figures, the teller sees that “x = + $50” and “y = – $70”. Then:

+ $z = + $50 – $70 = – $20

Obviously, the bank client was mistaken. His account balance is a debt (a negative amount by convention). We see an answer, which has a clear meaning, since the teller used an improper equality, due to an incorrect assumption that the client’s balance was a credit (a positive amount by convention).

Field of validity of an equality

The concept of equality, discussed in Chapter V, raises a question about the field of existence of a dependent variable in equality expressions.

Suppose we face the math expressions “y = √(x – 7)”, which actually means “+ y = √(+ x – 7)”. As previously stated, we may omit the plus (+) sign in front of the first term in any side of an equality, but not ignore it.

According to the New Math Model, “x” and “7” are absolute values. In order to have a proper equality, the value of “(+ x – 7)” on the right side of the equality must be an absolute value subject to the plus (+) sign, to be consistent with “+ y” on the left side of the equality. It means that “|x|” must be equal to or greater than “|7|”. This is the field of existence of “y”, the validity of that given math expression.

In case “|x|” is less than “|7|”, according to the Traditional Math Model we enter the field of complex numbers. Under the New Math Model, we deal with an improper equality, an unacceptable math equality.

These arithmetic and algebraic examples illustrate the logical thinking behind the

New Math Model (logical reasoning in math), and explain why algebra interprets the results of an improper equality as an imaginary responses (as a cube with a negative volume), an approach not accepted in the real world.

> **Algebra accepts the results of an improper equality, as a cube with a negative volume, but the real world (and the New Math Model) does not.**

Equations

I accepted in this book the understanding that an equation is an algebraic equality of a single unknown. An equation expresses a problem in mathematical language, and the resulting value for the unknown must be the answer to the equated problem The unknown value may result positive, negative or zero, but whatever the value, it must have a meaning.

As an example, suppose a writer wants to publish two different books, which have different publishing total costs (a fixed cost plus a variable cost, in a certain currency $), as follows:

Total cost of the first book, $C_{T1} = 500 + 30(x)$;

Total cost of the second book, $C_{T2} = 200 + 50(x)$;

Where "x" is the number of published books. The writer intends to invest around $ 20,000, and wants to receive the same quantity "x" of each book. Then:

$$C_{T1} + C_{T2} = 20{,}000$$

$$[500 + 30(x)] + [200 + 50(x)] = 20{,}000$$

$$80(x) = 19{,}300$$

$$x = 241.25$$

We see the writer will receive 241units of each book.

Interpretation of a Cartesian representation

I also introduced the understanding that the algebraic expressions of geometric figures placed in the Cartesian System are not equations, and do not have roots. As an example of such improper interpretation of algebraic equalities, consider the Cartesian formula of a parabola:

$$+ |y| = + |x|^2 - |2|.|x| - |3|$$

If we make it equal to zero, under the concepts of the Traditional Math Model we see the so-called 2^{nd}-degree equation:

$+ |x|^2 – |2|.|x| – |3| = 0$

In fact, as previously discussed, there is no equation. We are dealing with a well-known typical geometric figure (parabola) randomly placed in the Cartesian graph.

Notwithstanding that, once solved under the current math concepts (meaning the values of “x” for “y = 0”), we will get the two supposed roots, “+ x = – 1” and “+ x = + 3”, and both roots satisfy the given math expression, meaning “y = 0”.

Under my new propositions, I say that “+ x = + 3” is the correct answer[30], since it yields a proper equality, but “+ x = – 1” is a wrong answer, because it yields an improper equality. To demonstrate said statement I refer to the New Math Model (the different understanding of numbers and my Axiom), which imposes that not only the numbers “2” and “3” in the given math expression are absolute values, but also “y” and “x”.

It means that, to be correct responses, “x = |1|” and “x = |3|” must satisfy the algebraic expression being analyzed (obviously, we ignored “+ x = – 1”, since “+ |x| ≠ – |1|”). As illustrated in Figure A.1.

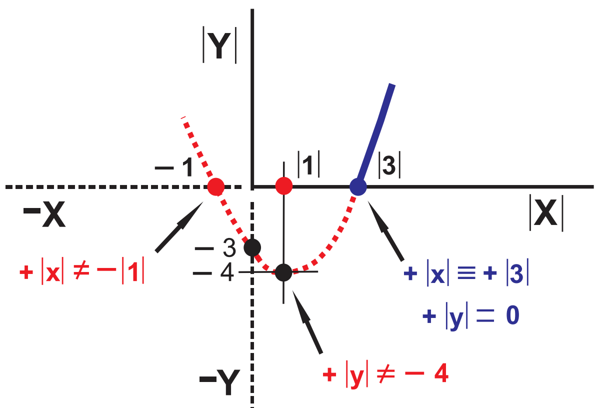
With the purpose to understand the difference, we need to remember that, as per the Traditional Math Model, before the variable “y” in the given expression there exist a hidden plus (+) sign. In algebra, we deal with positive values, negative values and absolute values[31], but the Cartesian graph does not contemplate absolute values. To be consistent, if we want to relate an algebraic equality with the Cartesian representation, we need to operate in the 1st quadrant or in the 3rd quadrant of the Cartesian graph (coordinates of “x” and “y” with equal signs).

If we test the two answers:

For “x = |3|”, “y = 0”, what means we have a proper equality;

For “x = |1|”, “y = – 4 ≠ 0”, what means we do not have an equality.

Figure A.1: Proper and improper equalities



30 “Correct answer” understood as the value of “x”, which makes “y = 0”.
31 We represent “values” with numbers, but they are not numbers. They mean natural things, as books, dollars and the like.

If we relate an algebraic equality to the Cartesian method, we cannot use the 2nd quadrant and the 4th quadrant of the Cartesian graph (coordinates of “x” and “y” with different signs) to avoid improper responses. That is why I say we must consider the variables in any algebraic expression as absolute values (modules). If we have a plus sign (+) before the value of “y”, we must also have a plus sign (+) before the value resulting from “x” in order to be dealing with a proper equality.

The concept of proper and improper equalities imposes the need to consider the plus (+) sign in front of all terms in algebraic expressions, which are assumed as belonging to the class of terms identified by said plus (+) sign. In other words, if we use the convention in force to omit the plus (+) sign in front of the first term in each side of an algebraic equality, we must remember that, under the premises of the New Math Model, said plus (+) sign exists, and we cannot ignore it.

The result yielded by an improper equality may have algebraic meaning, but it does not make sense in the real world. As an example, the volume of a cube equal to the difference between the volumes of two different cubes is never negative. It will always be the difference between the volume of the cube with a bigger volume and the volume of the cube with a smaller volume. A cube with a negative volume only exist in our imagination.

Rule of signs

As discussed in Chapter V, according to current concepts, we perform operations (multiplication, division and others) dealing with positive and negative numbers and terms formed with numbers and letters. In view of that, current math concepts state there exists a rule of signs under which an operation between numbers (or terms) of equal signs yields a result of positive sign, while an operation between numbers (or terms) of different signs yields a result of negative sign.

If we follow the concepts under the New Math Model, we do not need any rule of signs. We only perform algebraic sums (either addition or subtraction of isolated numbers or terms formed by numbers and letters, representing absolute values), as we usually do when dealing with algebraic sums.

The plus (+) sign before the first element tells us to keep the sign of the other element in the result, while the minus (–) sign before the first element tells us to change the sign of the other element in the result, whatever the sign in front of the other element. Addition and subtraction are the sole operating commandments of the plus (+) and minus (–) signs in any algebraic expression.

As an arithmetic example, determine the result of the following math expression:

$$+ z = (-\ 2^2)(+\ 3)$$

Under the concepts of the Traditional Math Model, we apply the rule of signs, we understand that math expression as "$+ z = (- 2^2)(+ 3) = (+ 4)(+ 3) = + 12$", since that math model imposes that the even power of a negative value is a positive value:

$$+ z = + 12$$

We see a proper equality, since "$+ |z| = + |12|$", but not necessarily a correct result.

The New Math Model only accepts addition and subtraction commandments, in a manner that the same math expression:

$$+ |z| = (- |2^2|)(+ |3|) = - (|4|)(+ |3|) = - (|4|)(|3|) = - |12|$$

$$+ |z| = - |12|$$

The New Math Model gives a different result, but we did not use any rule of signs (other than addition and subtraction), and are aware we deal with an improper equality, because "$+ |z| \neq - |z|$".

As an algebraic example, find the value of "k" in the random math expression "$k = 2x - x^2 + 16$", when "$x = - 3$".

Following the Traditional Math Model,

$$+ k = + 2(- 3) - 3^2 + 16 = - 6 + 9 + 16 = + 19$$

Under the New Math Model,

$$+ k = + 2(- 3) - 3^2 + 16 = - 6 - 9 + 16 = + 1$$

Similarly, if we have the math expressions below[32], the operations according to the New Math Model will be as follows:

$$+ r = + 5 + \sqrt{(- 4)} - \sqrt{(- 9)} = + 5 - \sqrt{4} + \sqrt{9} = + 5 - 2 + 3 = + 6$$

$$+ s = + \sqrt{(- x)} - \sqrt{(- y)} = - \sqrt{x} + \sqrt{y}$$

The Traditional Math Model uses a rule of signs to perform multiplication, division and other operations with positive and negative elements. The New Math Model only applies addition and subtraction commandments to operations with absolute values.

Under these abstract examples, the two math models may or may not yield the same result, since we perform the arithmetic or algebraic operation under different conceptual premises.

As previously stated, according to the concepts here suggested, we only deal with algebraic sums of absolute values of an element of a same nature (terms of an equality), which we group into two classes by convention. To distinguish one class from the other, we assign the plus (+) sign to the terms of one class, and the minus

32 I do not envisage any useful application of this type of operation. I maintained the example for consistency.

(–) sign to the terms of the other class. Following, we use the same operation (addition) to find the two totals, the sum of the terms with the positive (+) sign, and the sum of the terms with the negative (–) sign. Finally, we use the same plus (+) sign to perform the algebraic sum of the two classes, with different signs (which in fact is a subtraction operation of absolute values).

The plus (+) sign and the minus (–) sign in front of a number or letter (as a term of the math expression) only indicate to what class that term belongs: the class commanded by the plus (+) sign or the class commanded by the minus (–) sign.

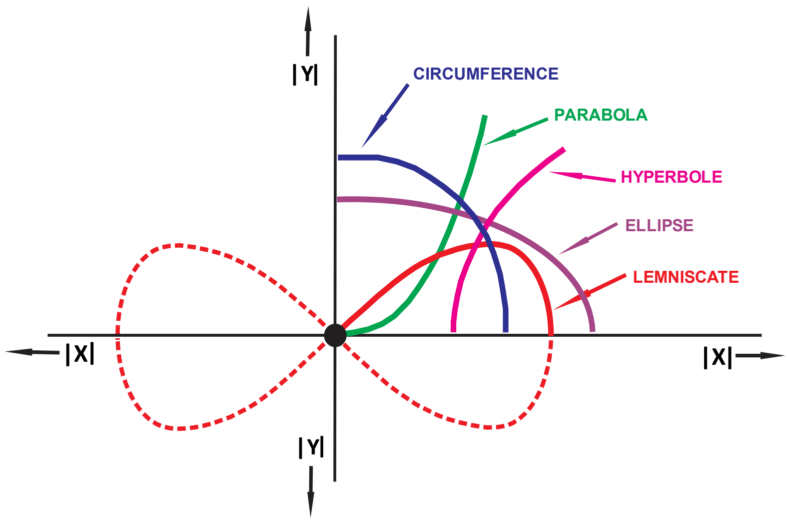
Cartesian graph

I previously stated that the Cartesian System is a tool to serve algebra and geometry. It is necessary to emphasize that, contrarily to general understanding, this tool has many limitations and we must handle it with care. As a limitation already mentioned, the Cartesian System deals with positive and negative coordinates, what means it does not contemplate absolute values. That is why we usually take positive values as equivalent to absolute values, but (as seen before) not always a valid approach. I tried to clarify this question when I introduced the Fundamental Axiom of Mathematics and the Alternative Cartesian System, the latter with coordinates as absolute values.

It is interesting to clarify that the use of positive and negative coordinates by the Cartesian method is a convention. According to the Author's Axiom and the Alternative Cartesian System (working with absolute values, which the traditional Cartesian System does not contemplate), we may mathematically define a geometric figure with the sole use of a single quadrant of the Alternative Cartesian System.

Figure A.2 shows a lemniscate and conical curves (ellipse, parabola, circumference and hyperbole), all of them placed and algebraically defined in the 1st quadrant of the Cartesian graph (the Alternative Cartesian System). We may, if we so wish, use the other quadrants of the graph to represent the entire figure (as we did with the lemniscate), but it is not necessary from the algebraic point of view.

Figure A.2: Geometric figures in the Alternative Cartesian graph



With the use of the Alternative Cartesian System, we understand the respective algebraic expressions, which work in all quadrants of the Cartesian graph (coordinates as absolute values), as follows:

Circumference:

$$|x|^2 + |y|^2 = |r|^2$$

Parabola:

$$|y| = |a|.|x|^2 + |b|$$

Ellipse:

$$|x|^2/|a|^2 + |y|^2/|b|^2 = |1|$$

Hyperbole:

$$|x|^2/|a|^2 - |y|^2/|b|^2 = |1|$$

Lemniscate:

$$(|x|^2 + |y|^2)^2 = 2(|a|^2)(|x|^2 - |y|^2)$$

I used the two vertical bars to indicate absolute values in all math terms simply to stress the concept. However, once the concept is accepted we may disregard the symbol, having in mind that we are dealing with absolute values, and that the plus (+) and minus (–) signs mean addition and subtraction operations, only.

System of equations

Under concepts accepted by the Traditional Math Model, we may face the problem of solving a system of equations, a set of "n" algebraic expressions, which involves “n” unknowns. The so-called system of equations may contain different algebraic expressions.

$$\begin{cases} \pm a_1z \pm b_1x \pm c_1y = \pm k_1 \\ \pm a_2z \pm b_2x \pm c_2y = \pm k_2 \\ \pm a_3z \pm b_3x \pm c_3y = \pm k_3 \end{cases}$$

When we solve a set of algebraic expressions as written above, we may find the coordinates “$x = x_i$”, “$y = y_i$” and “$z = z_i$” of a point “P_i” (if such a point actually exists), which satisfy all those math equalities.

Under the concepts of the New Math Model, except when we deal with a set of scientific formulas, which describes the behavior of meaningful phenomena, as we do in connection with physics subjects, the handling of the system of equations above referred is a dilettante occupation. Although a common exercise in math classes, the manipulation of random algebraic expressions is quite difficult to mean something in the real world.

The New Math Model limits the concept of equations to the algebraic sum (an algebraic equality) of a sole unknown, and the unknown value must be the answer to a question previously made. As an example:

$$\pm ax \pm b = \pm cx \pm d$$

To be a proper equality, all constants (“a”, “b”, “c” and “d”) and the variable (“x”) must be absolute values. Since we have a forced equality, if we isolate “x” in one side of the equality sign, the result of the other side may show the same sign of “x”, what means we have a proper equality. If the sign of the other sign is different from the sign of “x”, it means we deal with an improper equality. In both cases, and whatever that result, it must have a meaning, as the response to the problem represented by the equation. Whatever said value and whatever its sign, it must have a meaning. This is the rationale behind the unknown value in an equation.

As a result, and considering that we represent an equation by a single algebraic expression, which contains just one unknown, I do not envisage any practical situation under which we would have to handle a system of equations. In other words, under the concepts of New Math Model, we will not face a system of equations, as above mentioned (unless as ludic exercises or in connection with scientific formulas assumed as equations).

Irrational equation

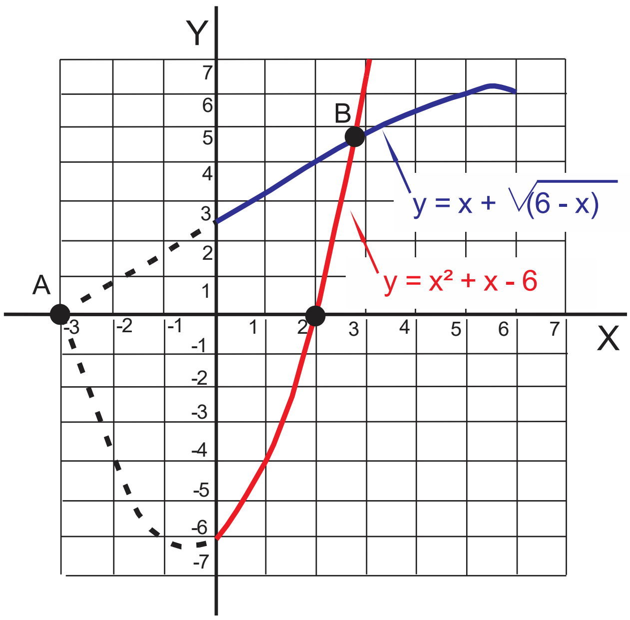
Going further with “equations”, let us analyze and comment a math expression, which the Traditional Math Model considers as being an irrational equation:

$$x + \sqrt{(6 - x)} = 0$$

Under the prevailing concepts, we solve that so-called irrational equation "$x + \sqrt{(6 - x)} = 0$" as equivalent to the expression "$x^2 + x - 6 = 0$". Once solved, we find two roots: one root that satisfy the given math expression ($x_1 = -3$) and other root, which does not satisfy said math expression ($x_2 = +2$). Then, we say that "$x_2 = +2$" is a "strange root".

Under the approach suggested in this book, this exercise indicates we are dealing with two different geometric figures (and neither of them is an equation), represented by different math expressions, as illustrated in Figure A.3: "$y = x^2 + x - 6$" and "$y = x + \sqrt{(6 - x)}$".

Figure A.3: Irrational equation



The first math expression represents a parabola randomly placed in the Cartesian System (an approach not recommend under the New Math Model), and we see that, when we make it equal to zero ($x^2 + x - 6 = 0$), it intercepts the abscissas axis when "$x = +2$" and "$x = -3$". The second math expression represents a modified geometric figure, which equal to zero ($x + \sqrt{(6 - x)} = 0$), only intercepts the abscissas axis when "$x = -3$". The two curves have a second common point around "$x = +2.8$".

Under the premises of this book, this modified geometric figure only exists in the interval "$0 \leq x \leq +6$", as per the continuous line in Figure A.3 (yielding a proper equality). The pair of "x" and "y" values, "$x = +2$" and "$y = 0$", does not correspond to a point in the given geometric figure, as it actually (and theoretically) does in the parabola. According to the prevailing concepts, we would enter the realm of complex numbers for "$x > 6$", a theory not accepted under the New Math Model.

Math does not fully accepts the applied operation of going from the given math expression "$x + \sqrt{(6 - x)} = 0$" to the other, "$x^2 + x - 6 = 0$", it being the reason for the occurrence of a strange root, which in fact does not exist. That is an approach to solve the so-called irrational equation, but the equality between the two math expressions only occurs in respect of the two common points "A" and "B" in the geometric figures. The interpretation that "$x = + 2$" is a strange root of the math expression "$x + \sqrt{(6 - x)} = 0$" is a misunderstanding caused by the acceptance of an invalid assumption that we are dealing with two equal math expressions, "$x + \sqrt{(6 - x)} = 0$" and "$x^2 + x - 6 = 0$", representing a same geometric figure.

If analyzed according to the new concepts, and keeping in mind that: (i) there are not negative numbers; and (ii) the independent variable "x" is an absolute value, we easily see there is no "x" value that satisfies the expression "$x + \sqrt{(6 - x)} = 0$", since that curve does not touch the abscissa axis.

The math expression "$x + \sqrt{(6 - x)} = 0$" is not, as accepted under current concepts, an equation and does not have any strange root. At the most, it accepts the pair of "x" and "y" values, "$x = - 3$" and "$y = 0$", as a point that belongs to the geometric figure represented in the traditional Cartesian graph. Besides that, what could possibly be the meaning and practical use of that math expression?

Indeterminate forms

In math, we see references to "undefined value" and "indeterminate form". I do not want to enter into any semantic discussion, but we understand "undefined value" as a relationship to which there is no value to satisfy it, while "indeterminate form" means there is a value not clearly or directly seen.

The ratio between a real number "m", different from zero, and zero is an undefined value.

$(m/0) \quad \rightarrow \quad m \neq 0 \rightarrow \quad$ undefined

We also refer to certain relationships, as "0/0" and "∞/∞" as indeterminate forms.

$(0/0)$ and $(\infty/\infty) \quad \rightarrow \quad$ indeterminate forms

Sometimes we face indeterminate forms when dealing with limit of functions. Consider the limit question below:

Lim $[(x^2 - 4)/(x - 2)]$, when "x" tends to "2".

Under a direct application of "$x = 2$" we will get "0/0", and we say we have an indeterminate form.

$(0/0)$ = indeterminate form

As we know:

$$(x^2 - 4) = (x - 2)(x + 2)$$

What means:

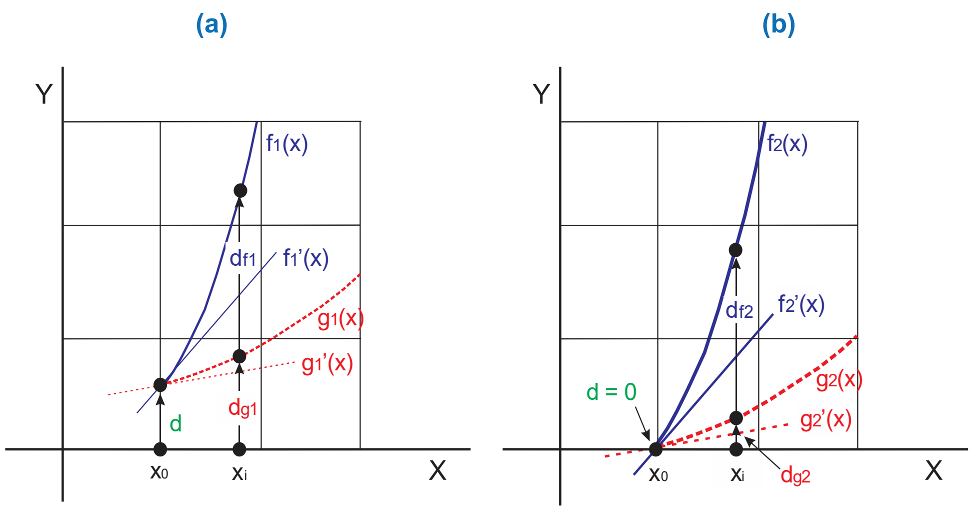
$$[(x^2 - 4)/(x - 2)] = [(x - 2)(x + 2)]/(x - 2) = (x + 2)$$

Actually, in the given expression, we multiply and divide the algebraic expression "$x + 2$" by the algebraic expression "$x - 2$", what means we multiply and divide "$x + 2$" by zero, when "$x = 2$".

Clearly, we want the limit of "($x + 2$)" when "x" tends to "2", which is "4".

Figure A.4 explains the situation with the help of the Cartesian System.

Figure A.4: Limit indeterminate form



As we see in Figure A.4(a), when "x" tends to any value (as "x_i"), the limit of "$f_1(x)/g_1(x)$" is the ratio of the respective ordinates of the two curves (full and dashed lines), "d_{f1}/d_{g1}", which is different from zero, when "d_{f1}" and "d_{g1}" are different from zero.

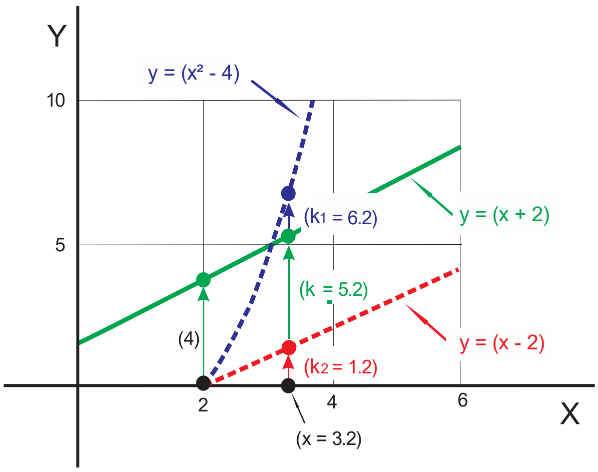
When "x" tends to a value as "x_0", a case in which "$d_{f1} = d_{g1} = d$", the ratio "d_{f1}/d_{g1}" is equal "d/d" and equal to "1". Then the limit of "$f_1(x)/g_1(x)$", when "x" tends to "x_0", is "1", an obvious result because, when "$x = x_0$", the two curves have a common point.

However, as in Figure A.4(b), if that common point stays on the abscissa axis ($x = x_0$), "$d_{f2} = d_{g2} = 0$", the ratio "d_{f2}/d_{g2}" is equal to "0/0", meaning the desired limit is an indeterminate form.

Figure A.5 illustrates the numerical examples above written:

Lim [($x^2 - 4$)/($x - 2$)], when "x" tends to "2".

Figure A.5: Numerical example of indeterminate form



When "$x = 3.2$", "$k_1/k_2 \neq 0$", since "$k_1 = 6.2$" and "$k_2 = 1.2$", what yields a ratio equal to "5.2", as the desired limit. When the value of "x" is a common point, which also is located in the reference line (the axis of the abscissas), as "$x = 2$", there is no way to find the limit. Since "$k_1 = k_2 = 0$", we need to take the proper algebraic expression, which is given by the continuous line "$y = x + 2$", and find "4" as the desired limit. In this numerical example, when "$x = 2$", there happens an indeterminate form.

Indeterminate form is a consequence of the poor understanding about algebraic expressions and the inadequate use of the Cartesian System, an improper use of algebraic expressions. There is an apparent indetermination, but we should not blame the number zero. We should blame algebra and its imperfections.

That is why under these circumstances we use the "Rule of L'Hospital" to find the desired limit: the limit of the given relationship "$f_2(x)/g_2(x)$" is equal to the limit of the ratio of the respective derivatives at that common point "$f_2'(x)/g_2'(x)$".

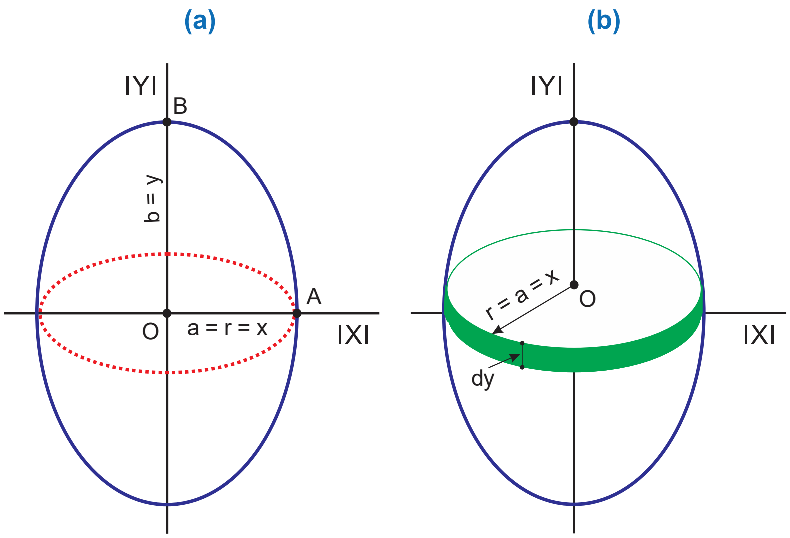
The example above shows that the indeterminate form is a consequence of the peculiarities of the algebraic expressions and their interaction with the Cartesian System.

In case of limits, we should not blame the number zero for the indeterminate form. We should care about the imperfections and misuse of algebra in connection with the Cartesian System.

Calculus

We may use a common example of calculus to find the volume of a regular ellipsoid formed by the semi-axes "a" and "b", as we see in Figure A.6. If we rotate the portion of the ellipse seen in the first Cartesian quadrant, "AOB", around the "Y" axis, following the circumference with center in "O" and radius "$r = a = x$", we build a half ellipsoid.

Figure A.6: Volume of an ellipsoid



Consider the cylinder of radius "r = a = x", and an infinitesimal height "dy". The volume of said infinitesimal cylinder will be:

$$dV = \pi r^2 dy = \pi x^2 dy$$

We also know we can define the value of "x^2" from the ellipse formula:

$$x^2/a^2 + y^2/b^2 = 1$$

$$x^2 = a^2(1 - y^2/b^2) = a^2 - a^2(y^2/b^2)$$

Then, by integrating the math expression between "0" and "b", and keeping in mind we are building half ellipsoid:

$$V_{1/2} = \int_0^b \pi x^2 dy = \int_0^b \pi a^2[1 - (y^2/b^2)]dy$$

$$V_{1/2} = (2/3)\pi a^2 b$$

$$V = (4/3)\pi a^2 b$$

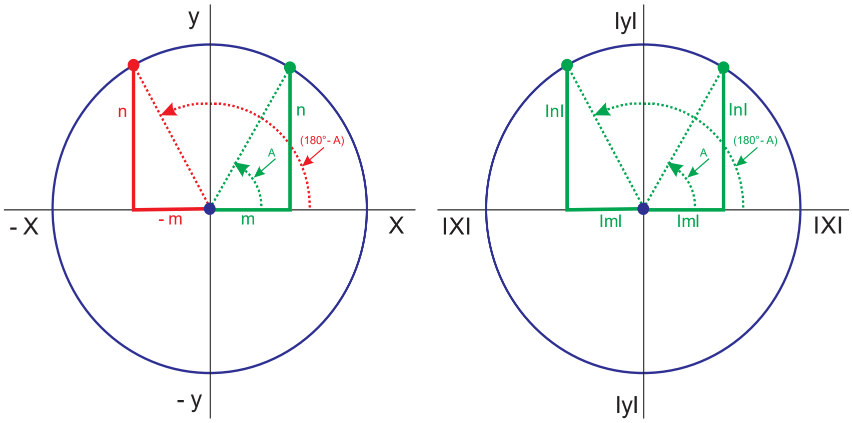
I used the New Math Model and it yielded the same useful result the Traditional Math Model would, because in both cases we deal with absolute values, not with positive and negative numbers and letters.

Trigonometry

As other example of the application of the New Math Model fundamentals to the math segments, I will consider the basic trigonometric functions, "sin (A) = n" and "cos (A) = m", as illustrated in Figure A.7(a).

Figure A.7: Trigonometry

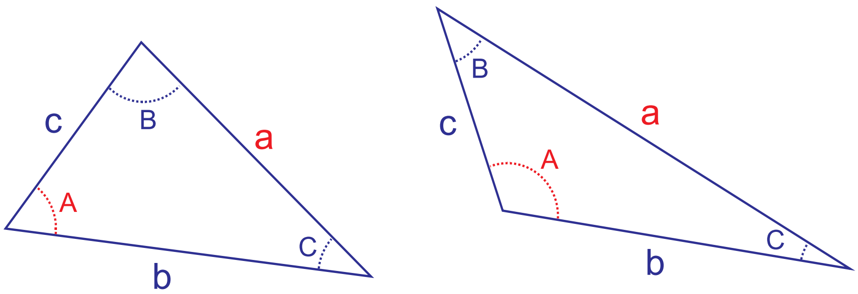
(a) Traditional Math Model — **(b)** New Math Model



I will deal with trigonometry in the same way I treated the algebraic expressions in the Cartesian System. Under the Traditional Math Model, “sin” and “cos” may be positive or negative. Under the New Math Model, “sin” and “cos” are absolute values. It means that, when the angle “A” varies from “0°” to “90°”, “sin (A)” varies from “0” to “1”, and “cos (A)” varies from “1” to “0”. We only need the 1st quadrant of the Cartesian graph.

As a practical example, let us consider a general triangle, as in Figure A.8. Suppose we want to find the value of “a” in function of the other triangle data (sides and angles).

Figure A.8: Triangles



Under the Traditional Math Model

(Angle «A» from 0° to less than 180°)
$a^2 = b^2 + c^2 - 2bc(\cos A)$

Under the New Math Model

(Angle «A» from 0° to 90°)
$a^2 = b^2 + c^2 - 2bc(\cos A)$

(Angle «A» greater than 90°, less than 180°)
$a^2 = b^2 + c^2 + 2bc(\cos A)$

We know that: i) the sum of the internal angles in any triangle is equal to “180°”; and (ii) the most general triangles show three different sides and three different angles, either the acute triangle (all angles less than “90°”) or the obtuse triangle (one angle greater than “90°”, but less than “180°”). It is also obvious that, in terms of absolute values, “cos (A)” is equal to “cos (180° – A)”, as we see in Figure A.7(b).

When dealing with the Traditional Math Model, since it accepts that “sin” and “cos” may assume positive or negative values, we may use a unique formula “$a^2 = b^2 + c^2 - 2bc(\cos$ “A”), no matter we deal with an acute or an obtuse triangle (in case of an obtuse triangle, the “cos (A)” is negative, what changes the formula). According to the New Math Model, “sin” and “cos” are absolute values, then we have to show the difference between acute triangle, $a^2 = b^2 + c^2 - 2bc(\cos$ “A”), and obtuse triangle, $a^2 = b^2 + c^2 + 2bc(\cos$ “A”), in their respective formulas. As expected, the useful results would be the same.

In summary, when dealing with trigonometry, we will only use the 1st quadrant of the Cartesian graph. Trigonometric functions may appear as terms commanded by positive or by negative signs in any math expression, but they will always be absolute values. Obviously, it will be necessary to adjust the formulas accordingly. In seems that trigonometric functions and formulas will follow the Fundamental Axiom of Mathematics.

Logarithms

By definition, the logarithm of a number “N” is the power exponent “x” we need to raise a base “A” to obtain such number “N”. We say that “x” is the logarithm of “N” in base “A”. In terms of algebra,

$$A^x = N \qquad \rightarrow \qquad x = \log_A (N)$$

We see why a negative number does not have logarithm, and why the logarithm of zero is an undefined value. It is not possible to obtain a negative number nor a zero value from a power “A^x”. As I suggested in this book, even if the power “A^x” appears in an algebraic expressions as “$+ A^x$” or as “$- A^x$”, nevertheless, “A” and “x” remain as absolute values (“$+ |A^x|$” or as “$- |A^x|$”).

Newton’s binomial formula

Newton’s binomial formula states that:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

Consider the determination of “$k = (a - b)^n$”. To make it simpler, let us assume that “$a = 2$”, “$b = 3$”, and “$n = 2$”. Then we have:

$$k = (2 - 3)^2$$

Under the concepts of the Traditional Math Model, we know that "$k = (2 - 3)^2 = -1^2 = +1$".

Following Newton's binomial formula, we obtain the same incorrect result "$k = +1$":

$$k = (1)(2^2)(-3^0) + (2)(2)(-3) + (1)(2^0)(-3^2) = (+4) + (-12) + (+9) = +1$$

It seems obvious that we cannot have a positive result (as a credit) when we raise a negative value (as a debt) to a second power.

If we use the New Math Model (absolute values), we will see a different result:

$$k = (2 - 3)2 = (-1)^2 = -(1)^2 = -1$$

If we use Newton's binomial formula, we also get the same correct result, provided we introduce the required correction to Newton's binomial formula. For the same reasons we had to correct the trigonometric formulas relating to triangles (different concept of numbers and the related consequences), it is necessary to change the binomial algebraic sum from addition (+) to subtraction (–) in front of the numbers commanded by the minus (–) sign, and raised to odd exponents. Numbers commanded by the minus (–) sign, and raised to even exponents already change their signs, due to the premises of the New Math Model.

$$(x \pm a)^n = \sum_{k=0}^{n} \square \pm \binom{n}{k} x^k a^{n-k}$$

$$k = (1)(2^2)(-3^0) - (2)(2)(-3) + (1)(2^0)(-3^2) = (-4) - (-12) + (-9)$$

$$k = -4 + 12 - 9 = -1$$

Other math tools and segments

The math models we just compared, the Traditional Math Model and the New Math Model, adopt different approaches regarding some basic concepts, particularly the meaning and use of numbers and letters in algebraic expressions, as well as the way to perform certain operations.

Except in case of specific operations, as the even power of an isolated number or term commanded by the minus (–) sign[33], the useful results obtained will be the same, whatever the math model we apply, particularly when we use certain math tools not dependent upon algebraic concepts, as probability and combinatory analysis.

We see discrepancies when we deal with matters, which depend on certain algebraic concepts, as limits, trigonometry, analytic geometry, and with formulas, as

33 Meaningless operations under the premises of the New Math Model.

trigonometric formulas applicable to triangles, Newton’s binomial formula and others, which imply adjustments. The New Math Model avoids improper equalities, which yield strange, imaginary or even mistaken answers, as well as the required odd explanations.

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ABOUT THE AUTHOR

SANDOVAL AMUI

was born on March 03, 1939, in Sacramento, Minas Gerais, Brazil. He is a civil engineer, who holds a M.Sc. degree in petroleum engineering from Louisiana State University, with other graduate courses in offshore engineering from the University of Texas at Austin. He is also an attorney, and retired in 2020, after many years of work with international oil companies and law firms. During his long lasting carrier, he published articles and books on technical and legal subjects. Recently he has devoted his time to other personal enjoyable activities, including writing papers and books on mathematics and geometry. Mr. Amui emphasizes that he has never had mathematics as his professional field of endeavor.

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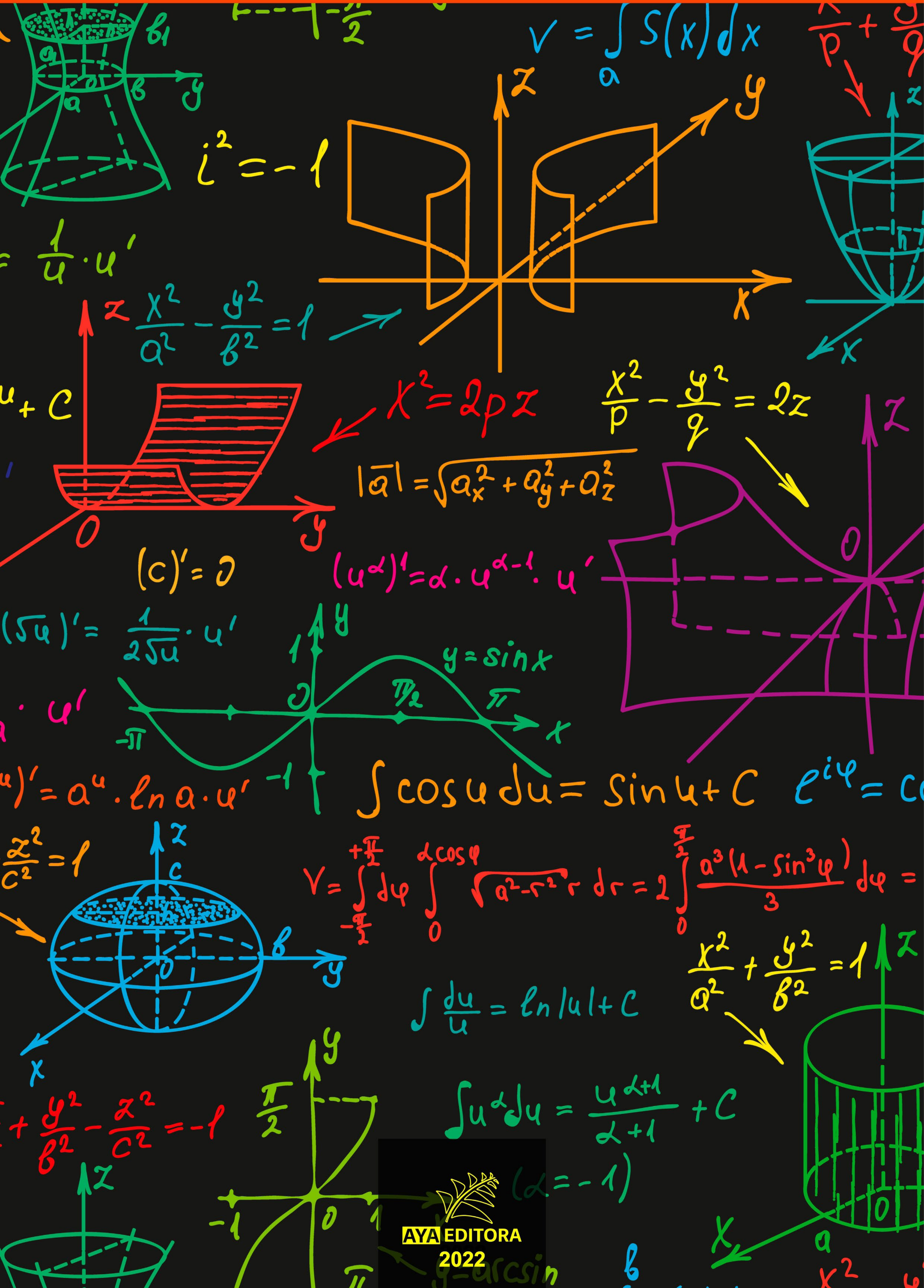

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